MADRAS CHRISTIAN COLLEGE HR. SEC. SCHOOL, CHETPET, CHENNAI – 31

IN HOC SIGNO

10TH STD

MATHEMATICS

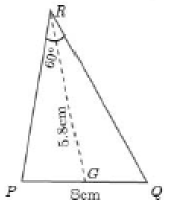
MINIMUM LEVEL STUDY MATERIAL

2019 – 2020

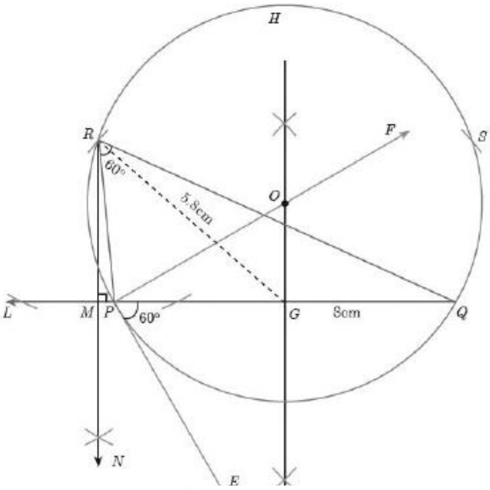
GEOMETRY

1. Construct a ΔPQR in which PQ = 8 cm, ∠R = 60^0 and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.

Solution :



Rough diagram



Construction

Step 1 : Draw a line segment PQ = 8cm.

Step 2 : At P, draw PE such that ∠QPE = 60^0.

Step 3 : At P, draw PF such that ∠EPF = 90^0.

Step 4 : Draw the perpendicular bisector to PQ, which intersects PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

Step 6 : From G mark arcs of radius 5.8 cm on the circle. Mark them as R and S.

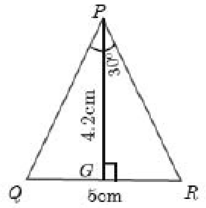
Step 7 : Join PR and RQ. Then ΔPQR is the required triangle .

Step 8 : From R draw a line RN perpendicular to LQ. LQ meets RN at M

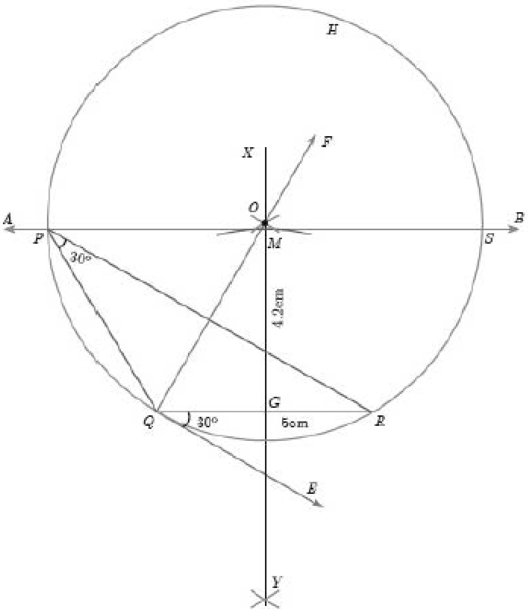
Step 9 : The length of the altitude is RM = 3.5 cm.

2. Construct a triangle ΔPQR such that QR = 5 cm, ∠P = 30^0 and the altitude from P to QR is of length 4.2 cm.

Solution :



Rough diagram



Construction

Step 1 : Draw a line segment QR = 5cm.

Step 2 : At Q, draw QE such that ∠RQE = 30^0.

Step 3 : At Q, draw QF such that ∠EQF = 90^0.

Step 4 : Draw the perpendicular bisector XY to QR, which intersects QF at O and QR at G.

Step 5 : With O as centre and OQ as radius draw a circle.

Step 6 : From G mark an arc in the line XY at M, such that GM = 4.2 cm.

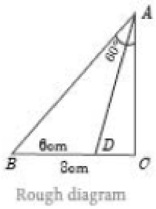
Step 7 : Draw AB through M which is parallel to QR.

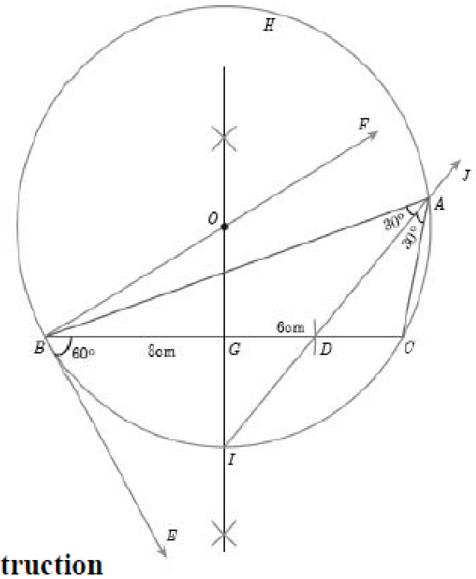
Step 8 : AB meets the circle at P and S.

Step 9 : Join QP and RP. Then ΔPQR is the required triangle.

3. Draw a triangle ABC of base BC = 8 cm, ∠A = 60^0 and the bisector of ∠A meets BC at D such that BD = 6 cm.

Solution :





Construction

Step 1 : Draw a line segment BC = 8cm.

Step 2 : At B, draw BE such that ∠CBE = 60^0.

Step 3 : At B, draw BF such that ∠EBF = 90^0.

Step 4 : Draw the perpendicular bisector to BC, which intersects BF at O and BC at G.

Step 5 : With O as centre and OB as radius draw a circle.

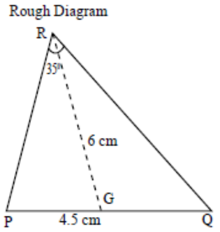
Step 6 : From B mark an arcs of 6 cm on BC at D.

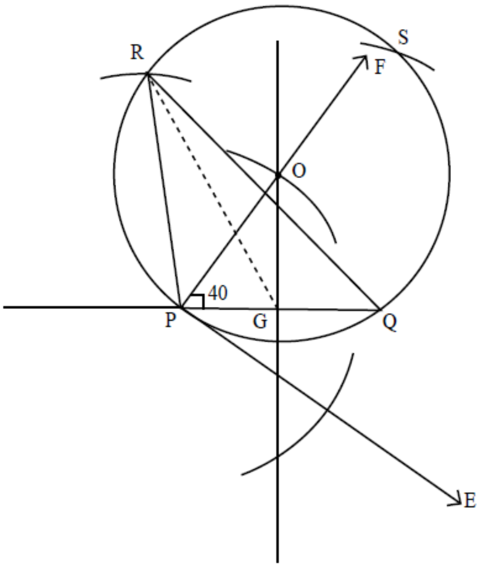
Step 7 : The perpendicular bisector intersects the circle at I. Join ID.

Step 8 : ID produced meets the circle at A. Now join AB and AC. Then ΔABC is the required triangle.

4. Construct a ΔPQR which the base PQ = 4.5 cm, ∠R = 35^0 and the median from R to PQ is 6 cm.

Solution :





Construction

Step 1 : Draw a line segment PQ = 4.5cm.

Step 2 : At P, draw PE such that ∠QPE = 35^0.

Step 3 : At P, draw PF such that ∠EPF = 90^0.

Step 4 : Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

Step 6 : From G mark arcs of 6 cm on the circle at RAS.

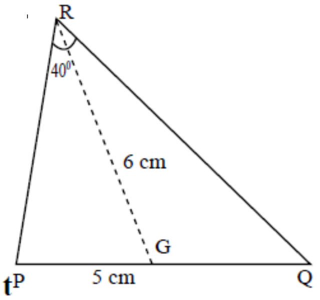
Step 7 : Join PR, RQ. Then ΔPQR is the required Δ.

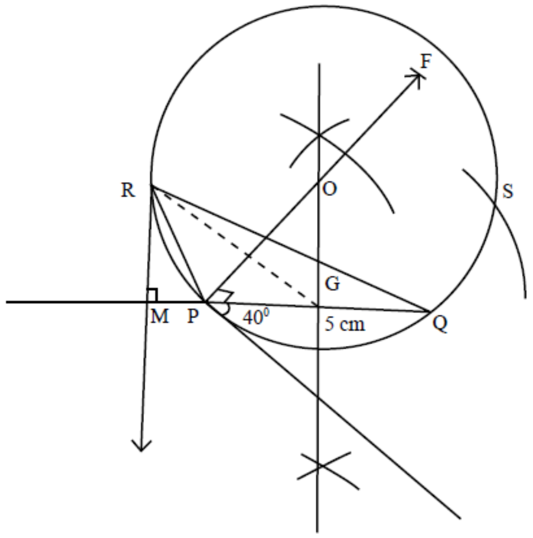
Step 8 : Join RG, which is the median.

5. Construct a ΔPQR in which PQ = 5 cm, $\angle P = 40^0$ and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

Solution :

Rough Diagram





Construction

Step 1 : Draw a line segment PQ = 5 cm.

Step 2 : At P, draw PE such that $\angle QPE = 40^0$.

Step 3 : At P, draw PF such that $\angle EPF = 90^0$.

Step 4 : Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

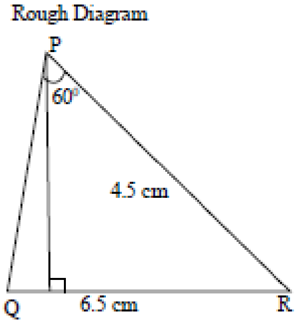
Step 6 : From G mark arcs of 4.4 cm on the circle radius 4.4m.

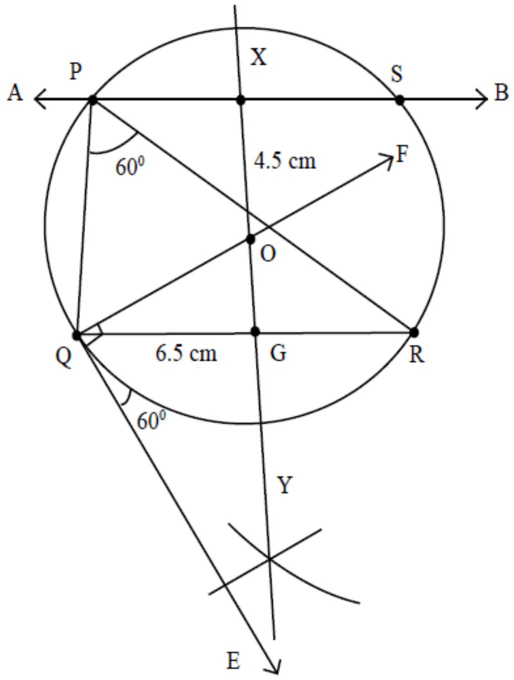
Step 7 : Join PR, RQ. Then ΔPQR is the required Δ.

Step 8 : Length of altitude is RM = 3 cm

6. Construct a ΔPQR such that QR = 6.5 cm, $\angle P = 60^0$ and the altitude from P to QR is of length 4.5 cm.

Rough Diagram





Construction

Step 1 : Draw a line segment QR = 6.5 cm.

Step 2 : At Q, draw QE such that $\angle RQE = 60^0$.

Step 3 : At Q, draw QF such that $\angle EQF = 90^0$.

Step 4 : Draw the perpendicular bisector XY to QR intersects QF at O & QR at G.

Step 5 : With O as centre and OQ as radius draw a circle.

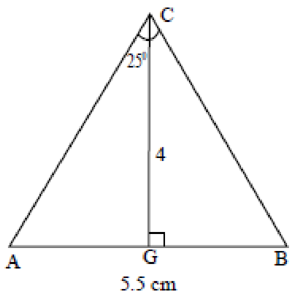
Step 6 : XY intersects QR at G. On XY, from G, mark arc M such that GM = 4.5 cm.

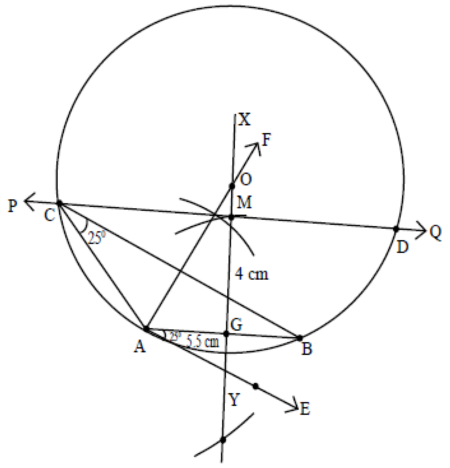
Step 7 : Draw AB, through M which is parallel to QR.

Step 8 : AB meets the circle at P and S.

Step 9 : Join QP, RP. Then ΔPQR is the required Δ.

7. **Construct a ΔABC such that $AB = 5.5$ cm, $\angle C = 25^0$ and the altitude from C to AB is 4 cm.**


Rough Diagram



Construction

Step 1 : Draw a line segment AB = 5.5 cm.

Step 2 : At A, draw AE such that $\angle BAE = 25^0$.

Step 3 : At A, draw AF such that $\angle EAF = 90^0$.

Step 4 : Draw the perpendicular bisector XY to AB intersects AF at O & AB at G.

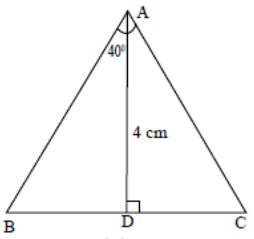
Step 5 : With O as centre and OA as radius draw a circle.

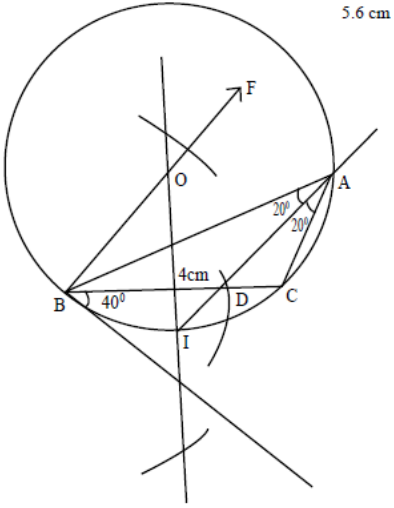
Step 6 : XY intersects AB at G. On XY, from G, mark arc M such that GM = 4 cm.

Step 7 : Draw PQ, through M parallel to AB meets the circle at C and D.

Step 8 : Join AC, BC. Then ΔABC is the required Δ.

8. **Draw a triangle ABC of base BC = 5.6 cm, $\angle A = 40^0$ and the bisector of $\angle A$ meets BC at D such that CD = 4 cm.**


Rough Diagram



Construction

Step 1 : Draw a line segment BC = 5.6 cm.

Step 2 : At B, draw BE such that $\angle CBE = 40^0$.

Step 3 : At B, draw BF such that $\angle CBF = 90^0$.

Step 4 : Draw the perpendicular bisector to BC meets BF at O & BC at G.

Step 5 : With O as centre and OB as radius draw a circle.

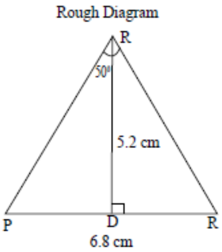
Step 6 : From B, mark an arc of 4 cm on BC at D.

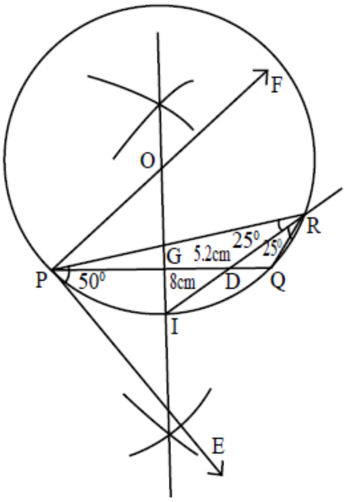
Step 7 : The ⊥r bisector meets the circle at I & Join ID.

Step 8 : ID produced meets the circle at A. Join AB & AC.

Step 9 : Then ΔABC is the required triangle.

9. **Draw ΔPQR such that $PQ = 6.8$ cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where $PD = 5.2$ cm.**





Construction

Step 1 : Draw a line segment $PQ = 6.8$ cm.

Step 2 : At P, draw PE such that $\angle QPE = 50^0$.

Step 3 : At P, draw PF such that $\angle QPF = 90^0$.

Step 4 : Draw the perpendicular bisector to PQ meets PF at O and PQ at G.

Step 5 : With O as centre and OP as radius draw a circle.

Step 6 : From P mark an arc of 5.2 cm on PQ at D.

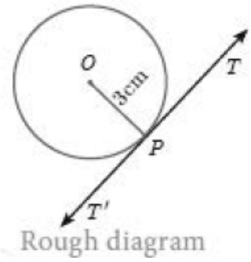
Step 7 : The perpendicular bisector meets the circle at R. Join PR and QR.

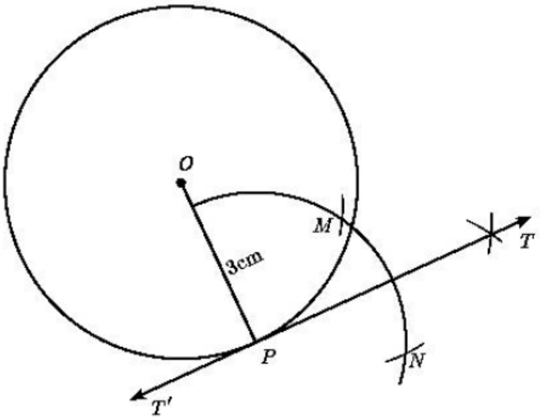
Step 8 : Then ΔPQR is the required triangle.

10. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution :

Given, radius $r = 3$ cm





Construction

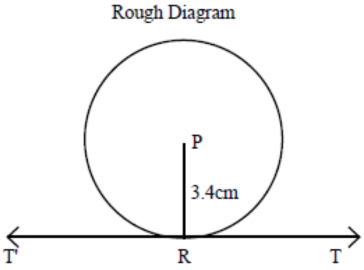
Step 1 : Draw a circle with centre at O of radius 3 cm.

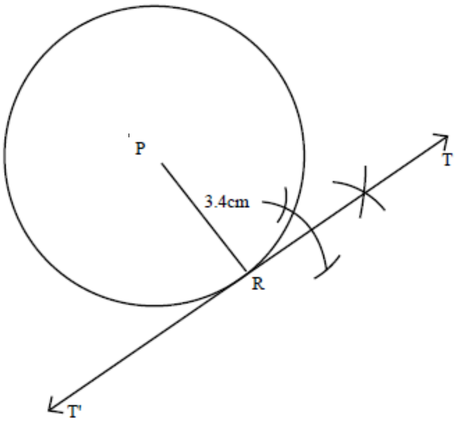
Step 2 : Take a point P on the circle. Join OP.

Step 3 : Draw perpendicular line TT' to OP which passes through P.

Step 4 : TT' is the required tangent.

11. **Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P ?**





Construction

Step 1 : Draw a circle with centre at P of radius 3.4 cm.

Step 2 : Take a point R on the circle and Join PR.

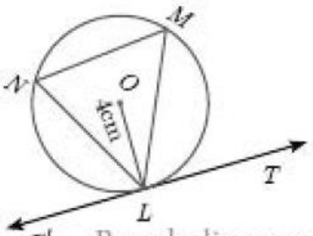
Step 3 : Draw perpendicular line TT' to PR which passes through R.

Step 4 : TT' is the required tangent.

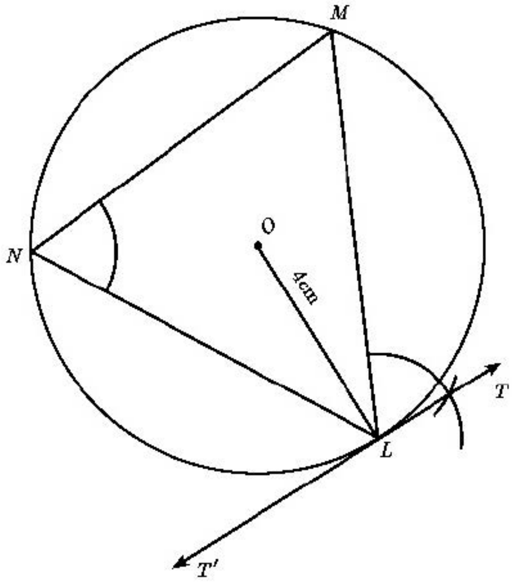
12 Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution :

Given, radius=4 cm



Rough diagram



Construction

Step 1 : With O as the centre, draw a circle of radius 4 cm.

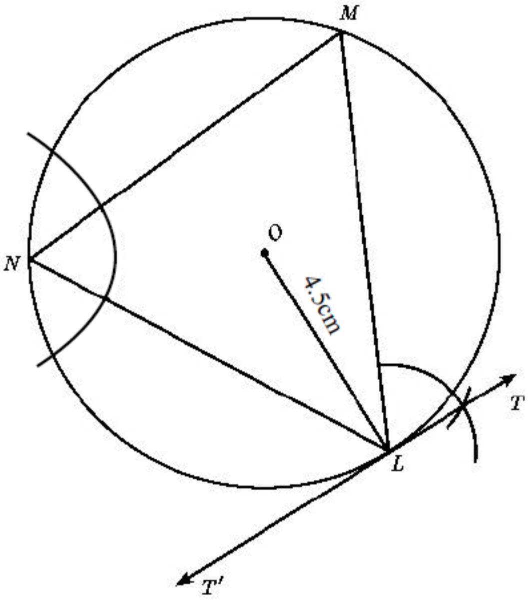
Step 2 : Take a point L on the circle. Through L draw any chord LM.

Step 3 : Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

Step 4 : Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.

Step 5 : TT' is the required tangent.

13. **Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.**



Construction

Step 1 : With O as the centre, draw a circle of radius 4 cm.

Step 2 : Take a point L on the circle. Through L draw any chord LM.

Step 3 : Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

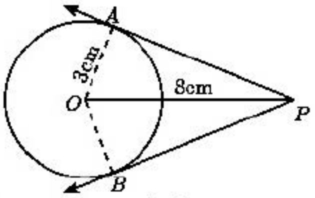
Step 4 : Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.

Step 5 : TT' is the required tangent.

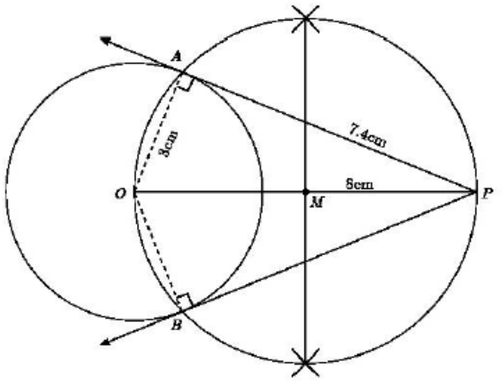
14. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Solution :

Given, diameter (d) = 6 cm, we find radius (r) = $\frac{6}{2}$ = 3 cm.



Rough diagram



Step 1 : With centre at O, draw a circle of radius 3 cm.

Step 2 : Draw a line OP of length 8 cm.

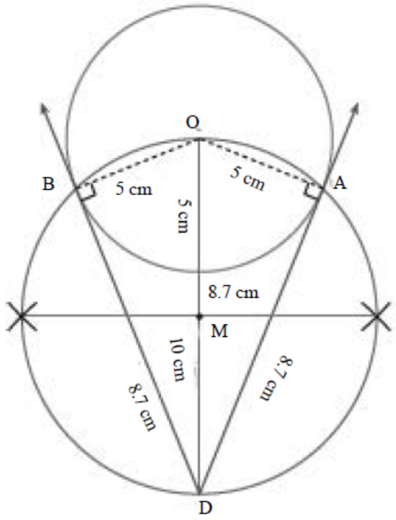
Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm.

15. **Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 5 cm.

Step 2 : Draw a line OP = 10 cm.

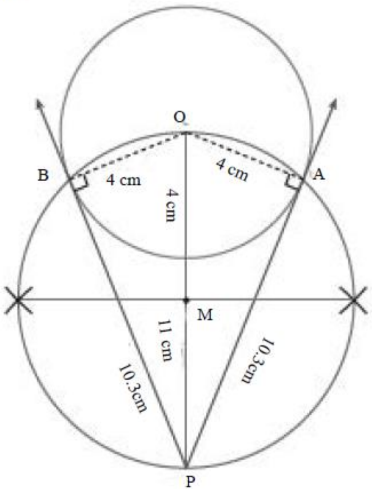
Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm.

16. **Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 4 cm.

Step 2 : Draw a line OP = 11 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. They are the required tangents AP = BP = 10.3 cm.

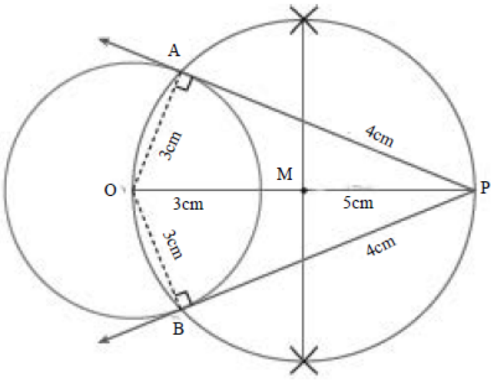
Verification : In the right angle triangle ΔOAP ,

$$\mathrm{AP} = \sqrt{\mathrm{OP}^2 - \mathrm{OA}^2}$$

$$= \sqrt{121 - 16} = \sqrt{105} = 10.3 \text{ cm}$$

17. **Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.**

Solution:



Construction

Step 1 : With centre at O, draw a circle of radius 3 cm. with centre at O.

Step 2 : Draw a line OP = 5 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step 5 : Join AP and BP. They are the required tangents AP = BP = 4 cm.

Verification :

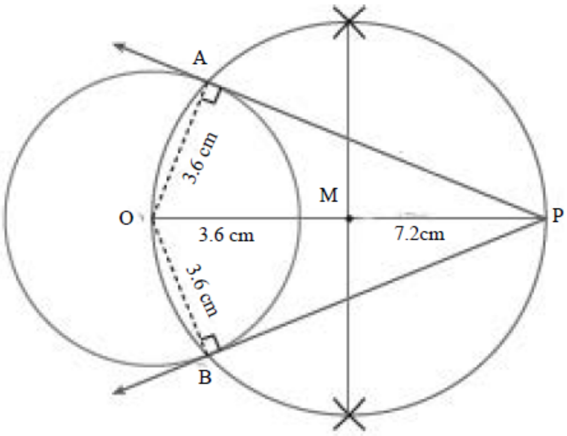
$$AP = \sqrt{\mathrm{OP}^2 - \mathrm{OA}^2}$$

$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4 \text{ cm}$$

18. **Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.**



Construction

Step 1 : Draw a circle of radius 3.6 cm. with centre at O.

Step 2 : Draw a line OP = 7.2 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts it M.

Step 4 : With M as centre and OM as radius, draw a circle which cuts previous circle at A and B.

Step5 : Join AP and BP. They are the required tangents AP = BP = 0.3 cm.

Verification :

$$AP = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{(7.2)^2 - (3.6)^2}$$
$$= \sqrt{51.84 - 12.96}$$
$$= \sqrt{38.88} = 6.3 \text{ (approx)}$$

Construction of similar triangles

Example 4.10

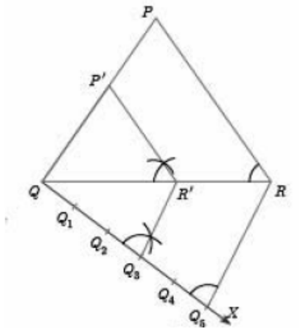
Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5}$<1)

Solution :

Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.

Steps of construction

1. Construct a ΔPQR with any measurement.



2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.

3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) points.

   Q_1, Q_2, Q_3, Q_4, and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$

4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q_5R to intersect QR at R'.

5. Draw line through R' parallel to the line RP to intersect QP at P'. Then, ΔP'QR' is the required triangle each of whose sides is three-fifths of the corresponding sides of ΔPQR.

20. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4}$>1)

Solution :

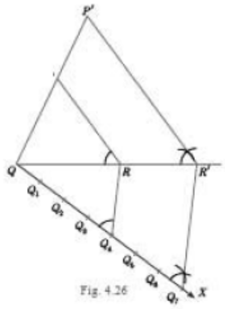


Fig 4.26

Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the triangle PQR.

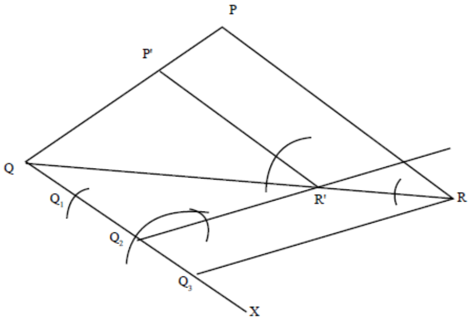
Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 7 (the greater of 4 and 7 in $\frac{7}{4}$) points.

   $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and Q_7 on QX so that

   $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 =$
4. Join Q_4 (the 4th point, 4 being smaller of 4 and 7 in $\frac{7}{4}$) to R and draw a line through Q_7 parallel to Q_4R, intersecting the extended line segment QR at R'.
5. Draw a line through R' parallel to RP intersecting the extended line segment QP at P' Then ΔP'QR' is the required triangle each of whose sides is seven-fourths of the corresponding sides of ΔPQR.

21. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$).**



Steps of construction

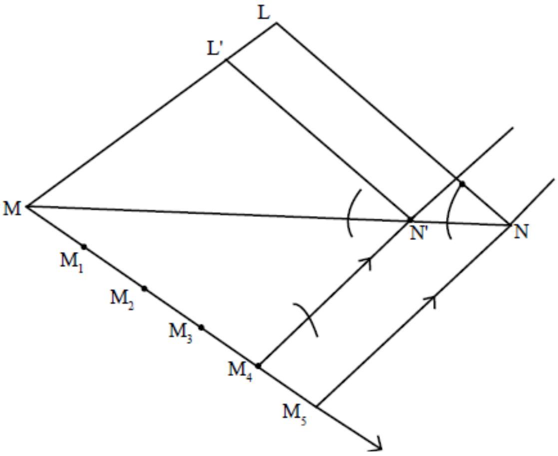
1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 3 points (greater of 2 and 3 in $\frac{2}{3}$) points.

   Q_1, Q_2, Q_3 on QX so that

   $QQ_1 = Q_1Q_2 = Q_2Q_3$
4. Join Q_3R and draw a line through Q_2 (3 being smaller of 2 and 3 in $\frac{2}{3}$) parallel to Q_3R to intersect QR at R'.
5. Draw line through R' parallel to the line RP intersecting the QP at P'. Then, ΔP'QR' is the required Δ.

22. **Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$**

Solution :



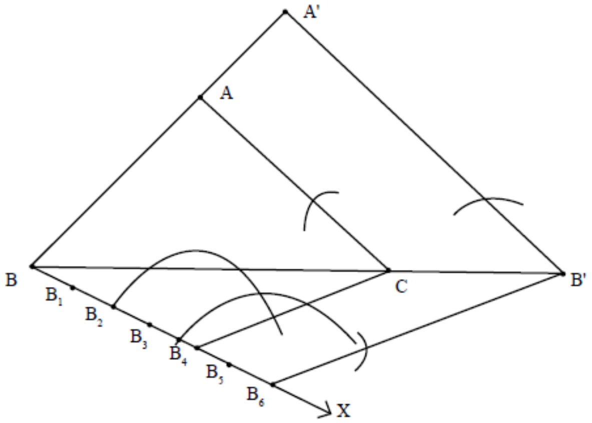
1. Construct a ΔLMN with any measurement.
2. Draw a ray MX making an acute angle with MN on the side opposite to vertex L.
3. Locate 5 points (greater of 4 and 5 in $\frac{4}{5}$) points.

   M_1, M_2, M_3, M_4 & M_5 so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$,
4. Join M_5 to N and draw a line through M_4 (4 being smaller of 4 and 5 in $\frac{4}{5}$) parallel to M_5N to intersect MN at N'.
5. Draw line through N' parallel to the line LN intersecting line segment ML to L'.

   Then, L'M'N' is the required Δ.

23. **Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5}$).**

Solution :



Steps of construction

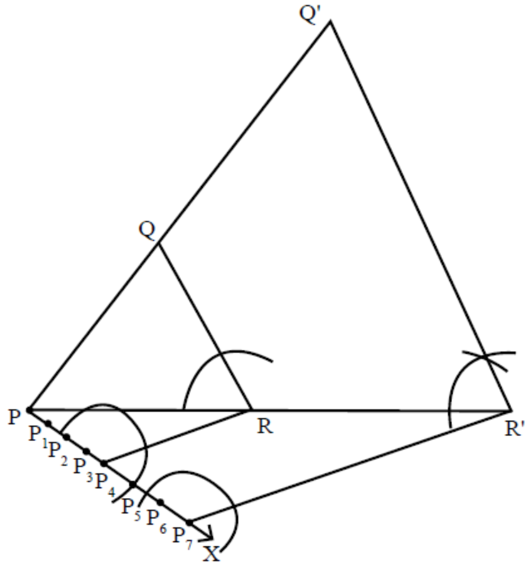
1. Construct a ΔABC with any measurement.
2. Draw a ray BX making an acute angle with BC on the side opposite to vertex A.
3. Locate 6 points (greater of 6 and 5 in $\frac{6}{5}$) points.

   $B_1, B_2, \ldots\ldots\ldots B_6$ on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_3B_4 = B_4B_5 = B_5B_6$,
4. Join B_4 (4 being smaller of 4 and 6 in $\frac{6}{4}$) to C and draw a line through B_6 parallel to B_4C to intersecting the extended line segment BC at C'.
5. Draw line through C' parallel to CA intersect the extended line segment BA to A'.

   Then, $\Delta A'B'C'$ is the required Δ.

24. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$).**

Solution :



Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray PX making an acute angle with PR on the side opposite to vertex Q.
3. Locate 7 points (greater of 3 and 7 in $\frac{7}{3}$) points.

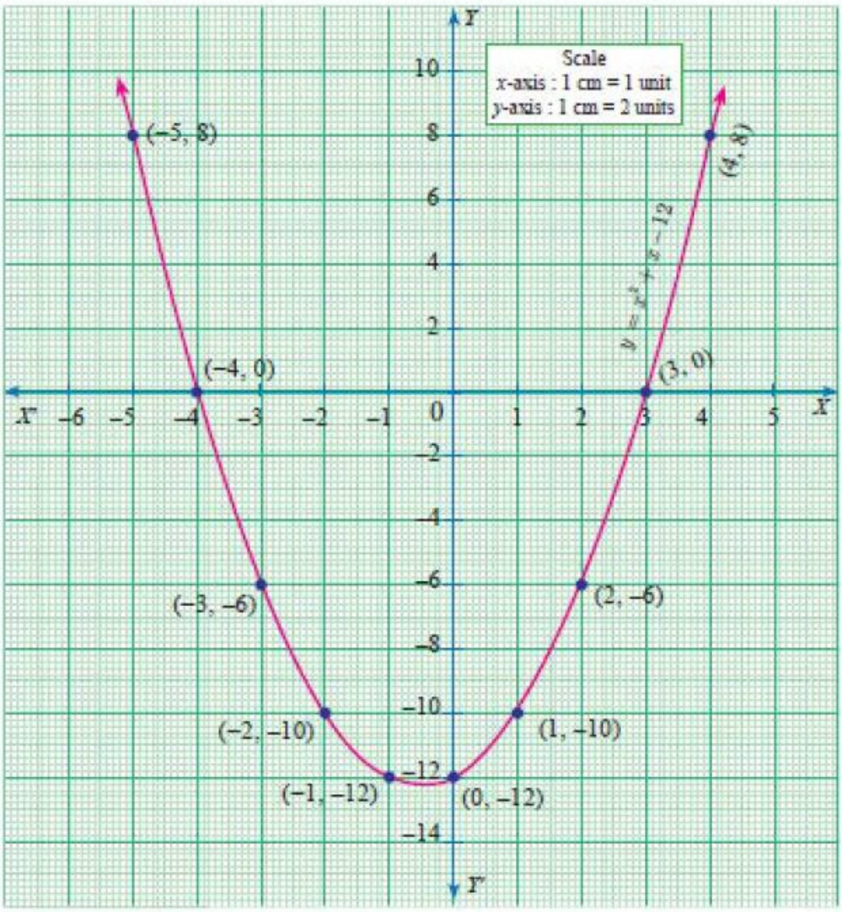
   $P_1, P_2, \ldots\ldots\ldots P_7$ on PX so that $PP_1 = P_1P_2 = P_2P_3 \ldots\ldots\ldots\ldots = P_6P_7$,
4. Join P_3R (3 being smaller of 3 and 7 in $\frac{7}{3}$) and draw a line through P_7 parallel to P_3R to intersecting the extended line segment PR at R'.
5. Draw line through R' parallel to QR intersect the extended line segment PQ to Q'.

   Then, $\Delta P'Q'R'$ is the required Δ.

GRAPH

1. *Discuss the nature of solution of the following quadratic equation $X^2 + X - 12 = 0$*

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
+ -	25 -17	26 -26	9 -15	4 -16	1 -13	0 -12	2 -12	6 -12	12 -12	20 -12	30 -12
Y	8	0	-6	-10	-12	-12	-10	-6	0	8	18

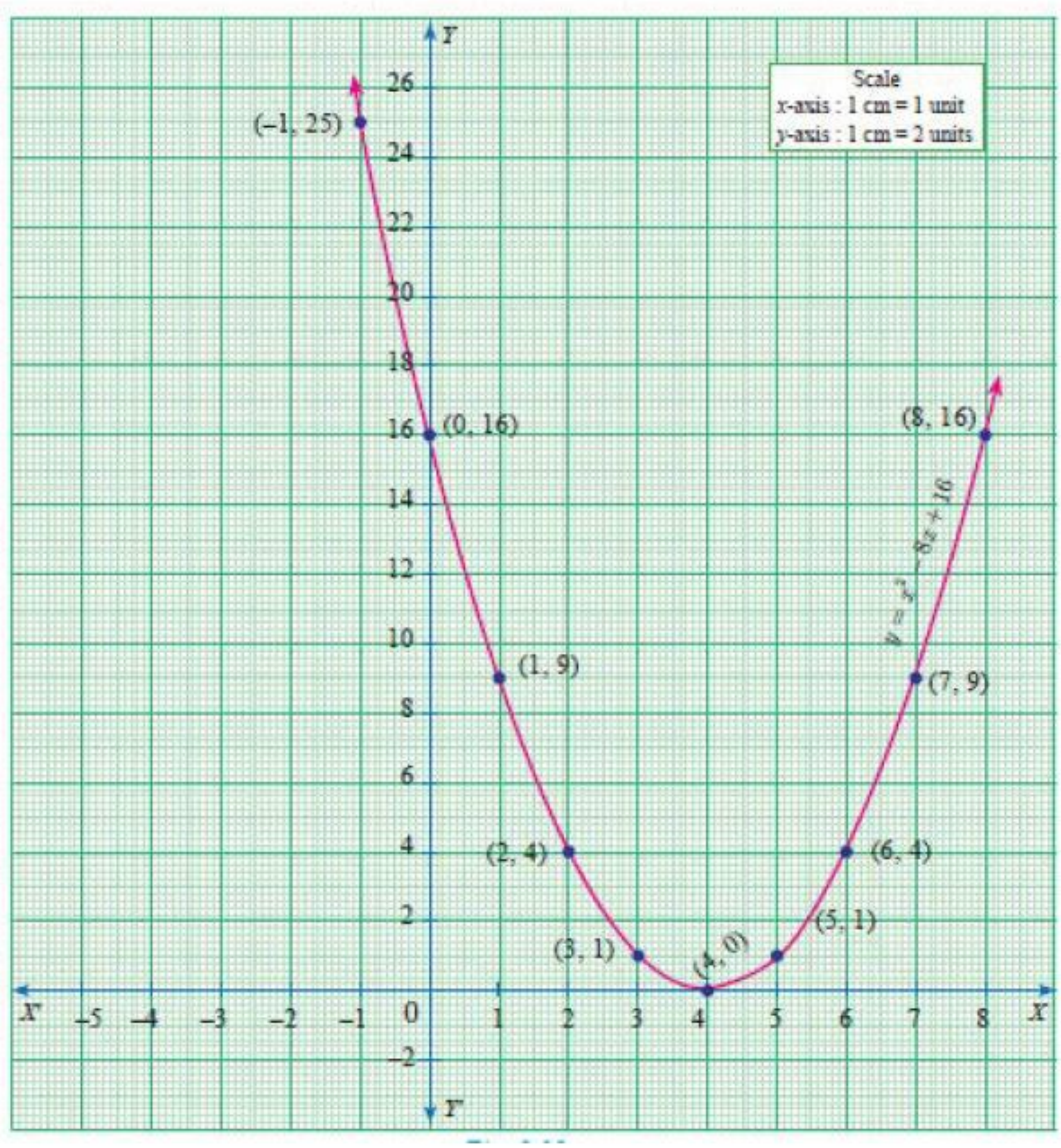


Solution set = { -4, 3 }

Therefore the roots are real and unequal.

2. Discuss the nature of solution of the following quadratic equation $X^2 - 8X + 16 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
-8X	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56
16	16	16	16	16	16	16	16	16	16	16	16	16	16
+ -	81 0	64 0	49 0	36 0	25 0	16 0	17 -8	20 -16	26 -24	32 -32	41 -40	52 -48	65 -56
Y	81	64	49	36	25	16	9	4	1	0	1	4	9

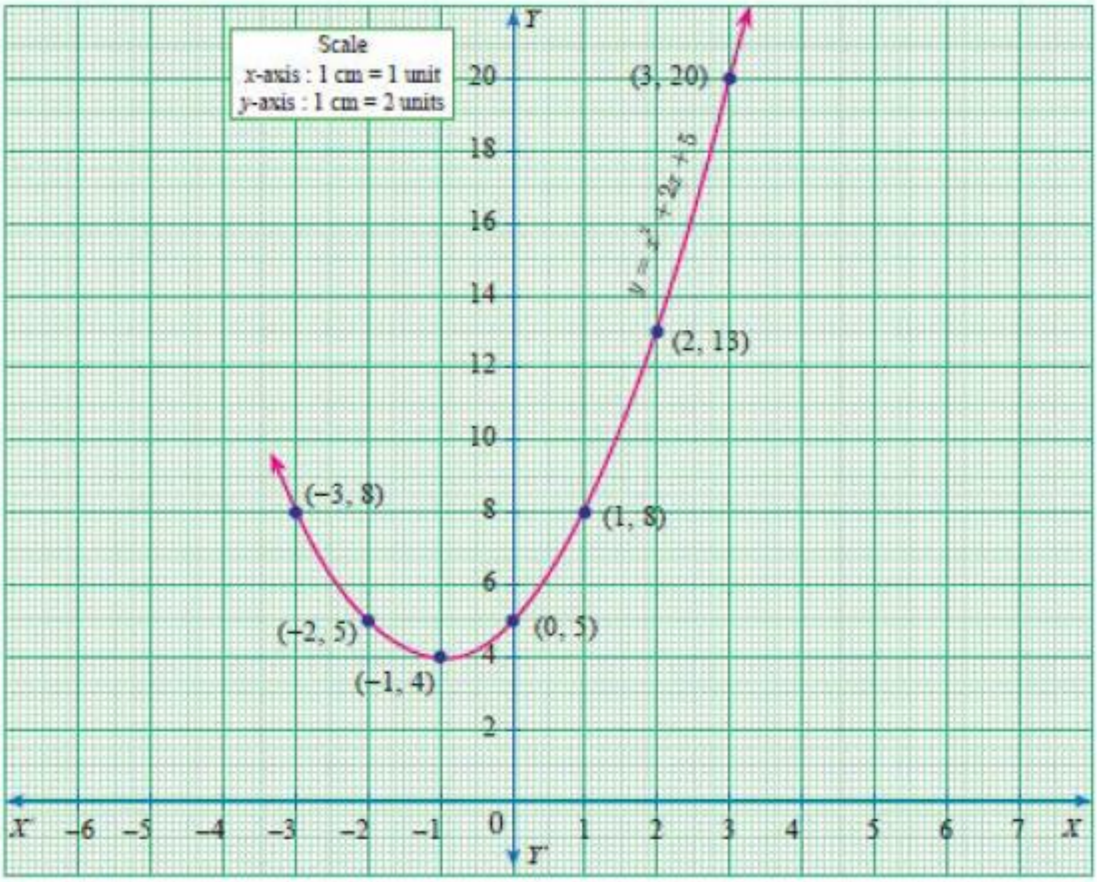


Solution set = {4, 4 }

Therefore the roots are real and equal.

3. ***Discuss the nature of solution of the following quadratic equation*** $X^2 + 2X + 5 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
2X	-10	-8	-6	-4	-2	0	2	4	6	8	10
5	5	5	5	5	5	5	5	5	5	5	5
+ -	30 -10	21 -8	15 -6	9 -4	6 -2	5 0	8 0	13 0	20 0	29 0	40 0
Y	20	13	8	5	4	5	8	13	20	29	40

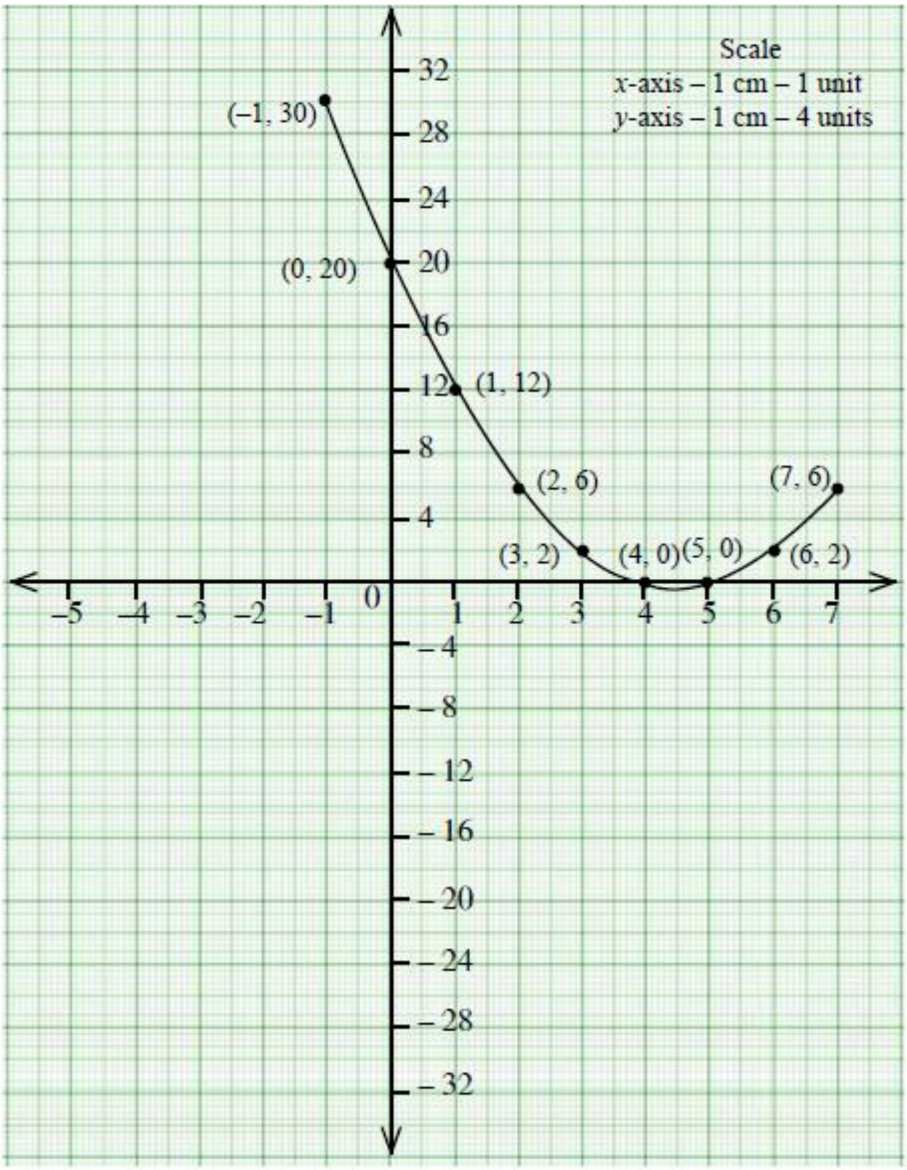


No solution

Therefore the roots are unreal.

4. Discuss the nature of solution of the following quadratic equation $X^2 - 9X + 20 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
-9X	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63
20	20	20	20	20	20	20	20	20	20	20	20	20	20
+ -	90 0	72 0	56 0	42 0	30 0	20 0	21 -9	24 -18	29 -27	36 -36	45 -45	56 -54	69 -63
Y	90	72	56	42	30	20	12	6	2	0	0	2	6

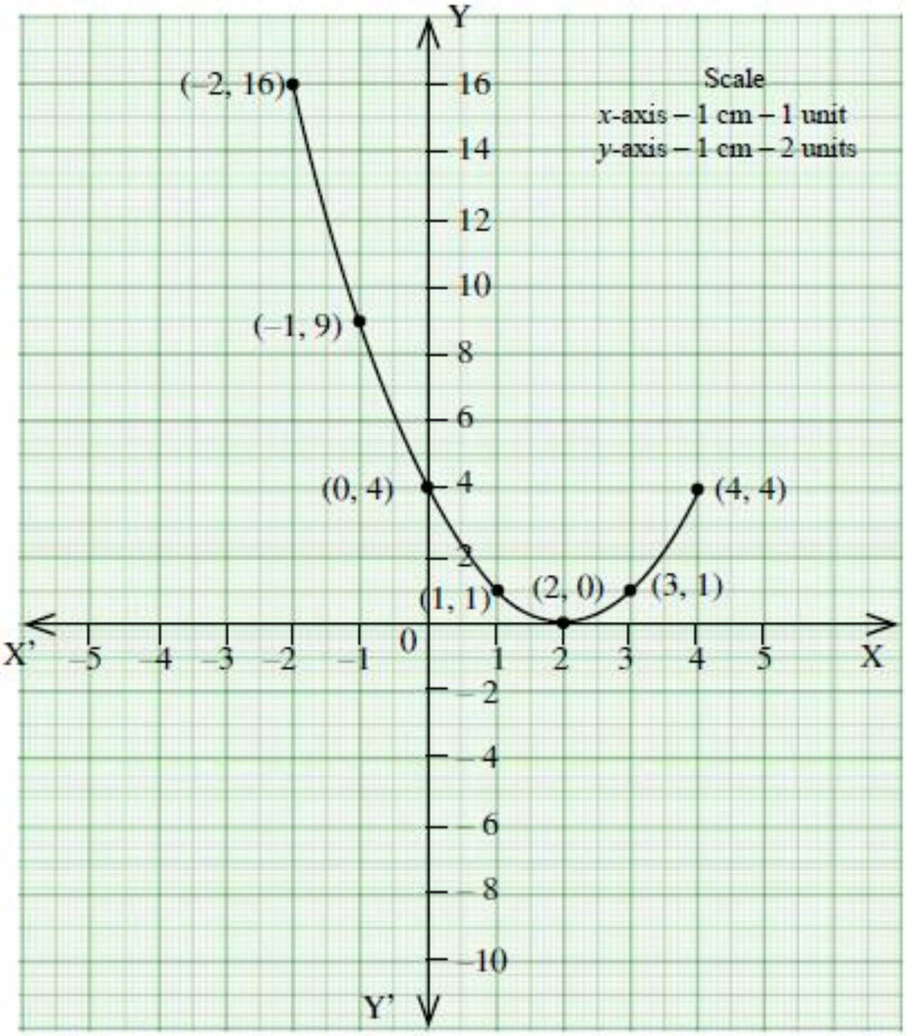


Solution : { 4, 5 }

Therefore the roots are real and unequal.

5. Discuss the nature of solution of the following quadratic equation $X^2 - 4X + 4 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-4X	20	16	12	8	4	0	-4	-8	-12	-16	-20
4	4	4	4	4	4	4	4	4	4	4	4
+ -	49 0	36 0	25 0	16 0	9 0	4 0	5 -1	8 -8	13 -12	20 -16	29 -25
Y	49	36	25	16	9	4	1	0	1	4	9

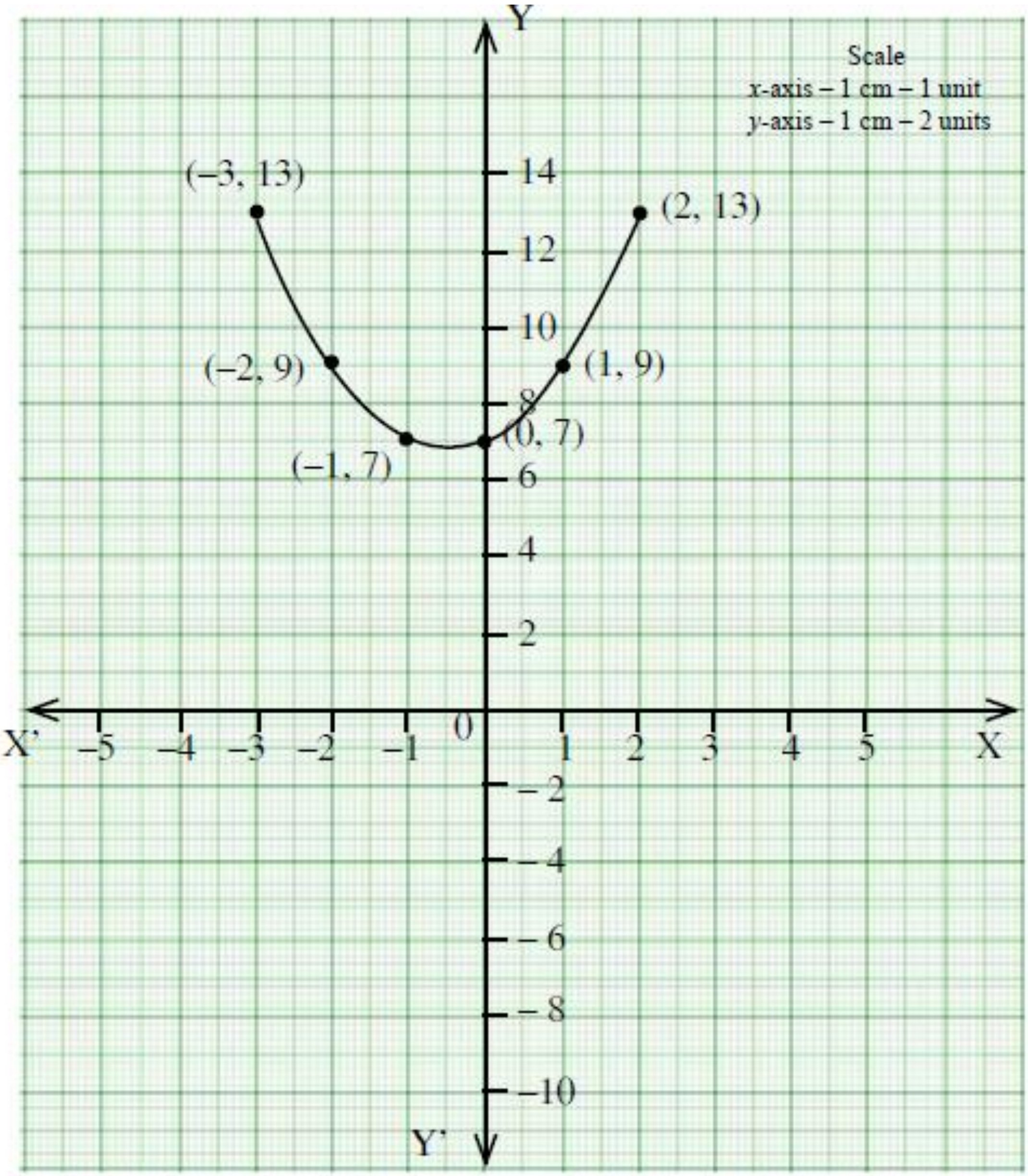


Solution : { 2, 2 }

Therefore the roots are real and equal

6. Discuss the nature of solution of the following quadratic equation $X^2 + X + 7 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
7	7	7	7	7	7	7	7	7	7	7	7
+ -	32 -5	23 -4	16 -3	11 -2	8 -1	7 0	9 0	13 0	19 0	27 0	37 0
Y	27	19	13	9	7	7	9	13	19	27	37

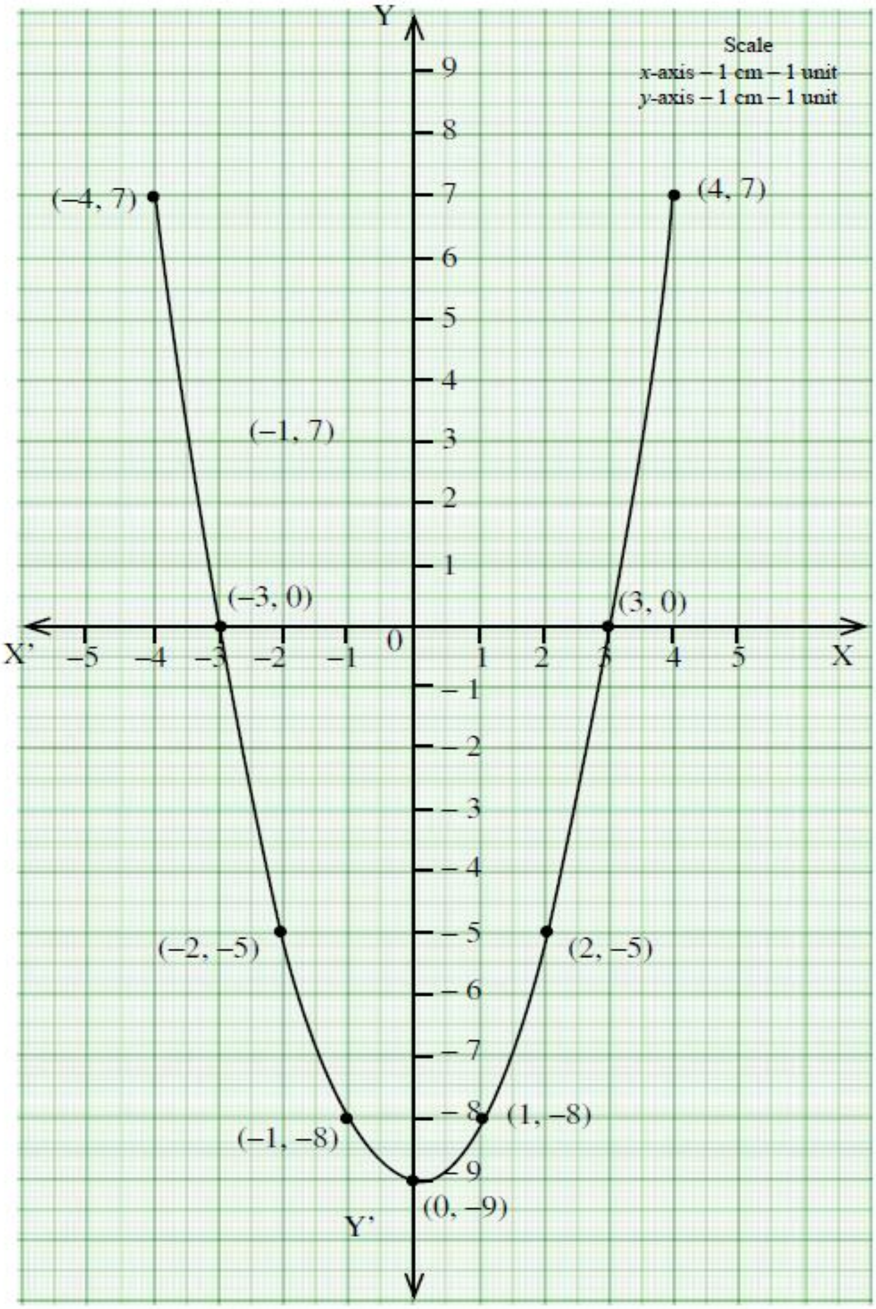


No Solution

Therefore the roots are unreal.

7. Discuss the nature of solution of the following quadratic equation $X^2 - 9 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
Y	16	7	0	-5	-8	-9	-8	-5	0	7	16



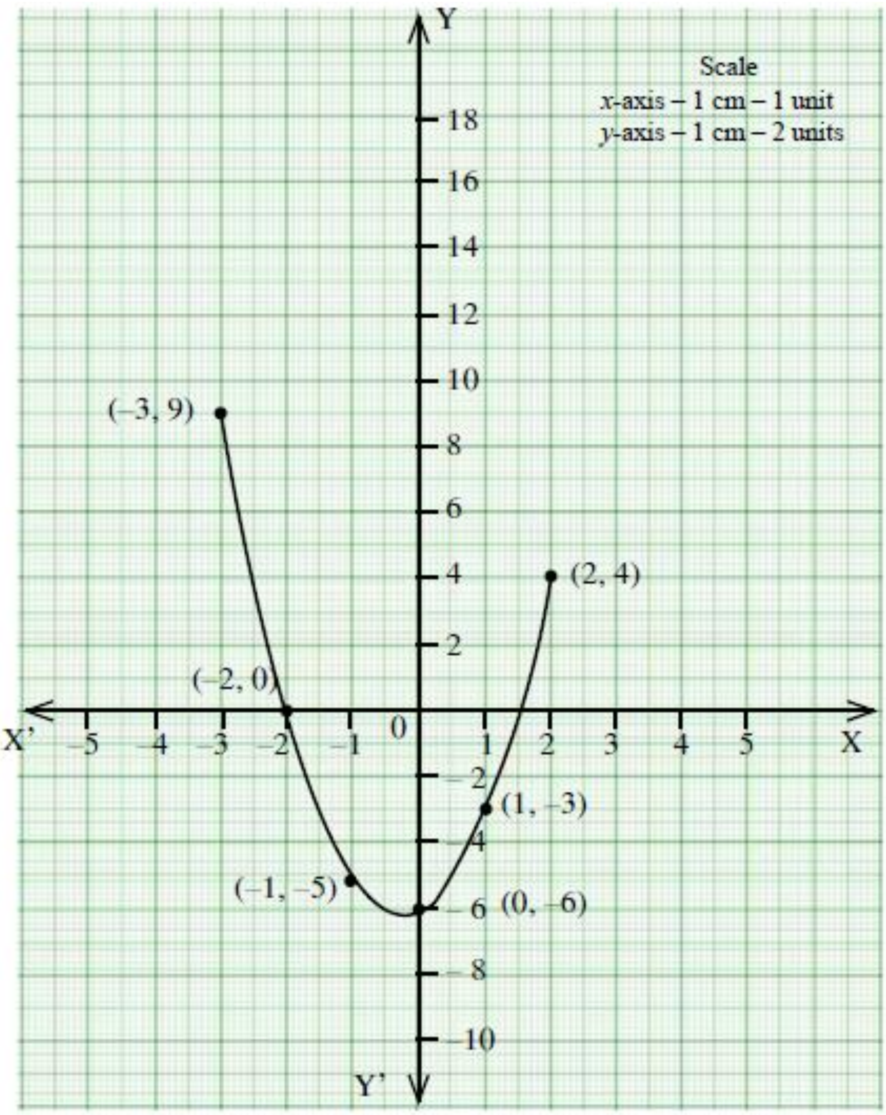
Solution : { -3, 3 }

Therefore the roots are real and unequal.

8. *Discuss the nature of solution of the following quadratic equation $(2x - 3)(x + 2) = 0$*

$(2x - 3)(x + 2) = 0$ $\Rightarrow 2x^2 + x - 6 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
$2X^2$	50	32	18	8	2	0	2	8	18	32	50
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
+ -	50 -11	32 -10	18 -9	8 -8	2 -7	0 -6	3 -6	10 -6	21 -6	36 -6	
Y	39	22	9	0	-5	-6	-3	4	15	30	49



Solution : { -2, 1.5 }

Therefore the roots are real and unequal.

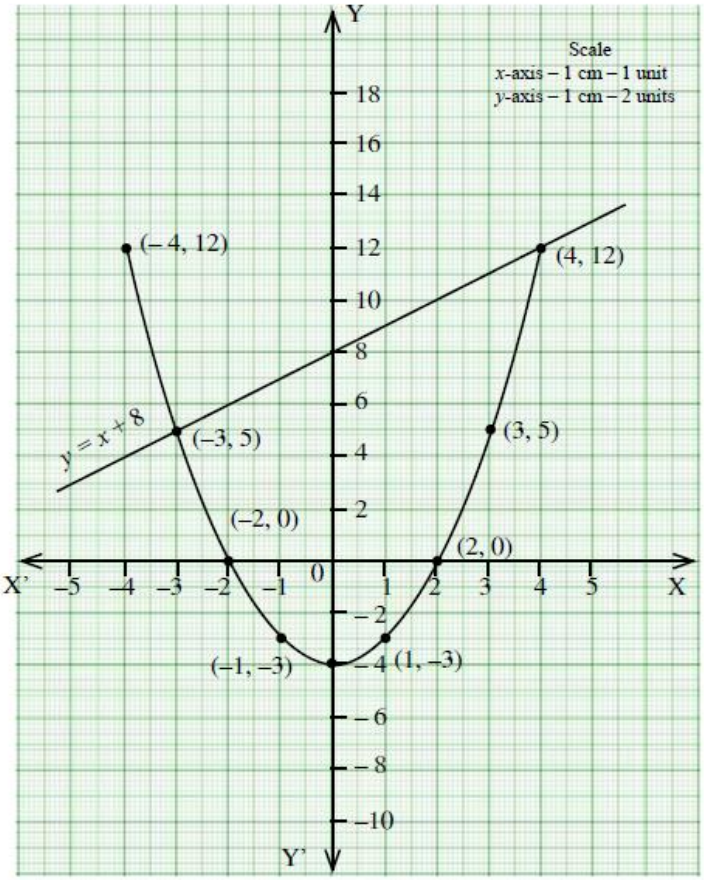
9. Draw the graph of $Y = X^2 - 4$ and hence solve $X^2 - X - 12 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
Y	21	12	5	0	-3	-4	-3	0	5	10	21

To solve $x^2 - x - 12 = 0$, subtract $x^2 - x - 12 = 0$ from $y = x^2 - 4$.

$$\begin{array}{rl} \text{from} & y = x^2 - 4 \\ & y = x^2 + 0x - 4 \\ & \underline{0 = x^2 - x - 12} \\ & y = x + 8 \end{array}$$

x	-4	-3	-2	-1	0	1	2	3	4
y	4	5	6	7	8	9	10	11	12



Solution : { -3, 4 }

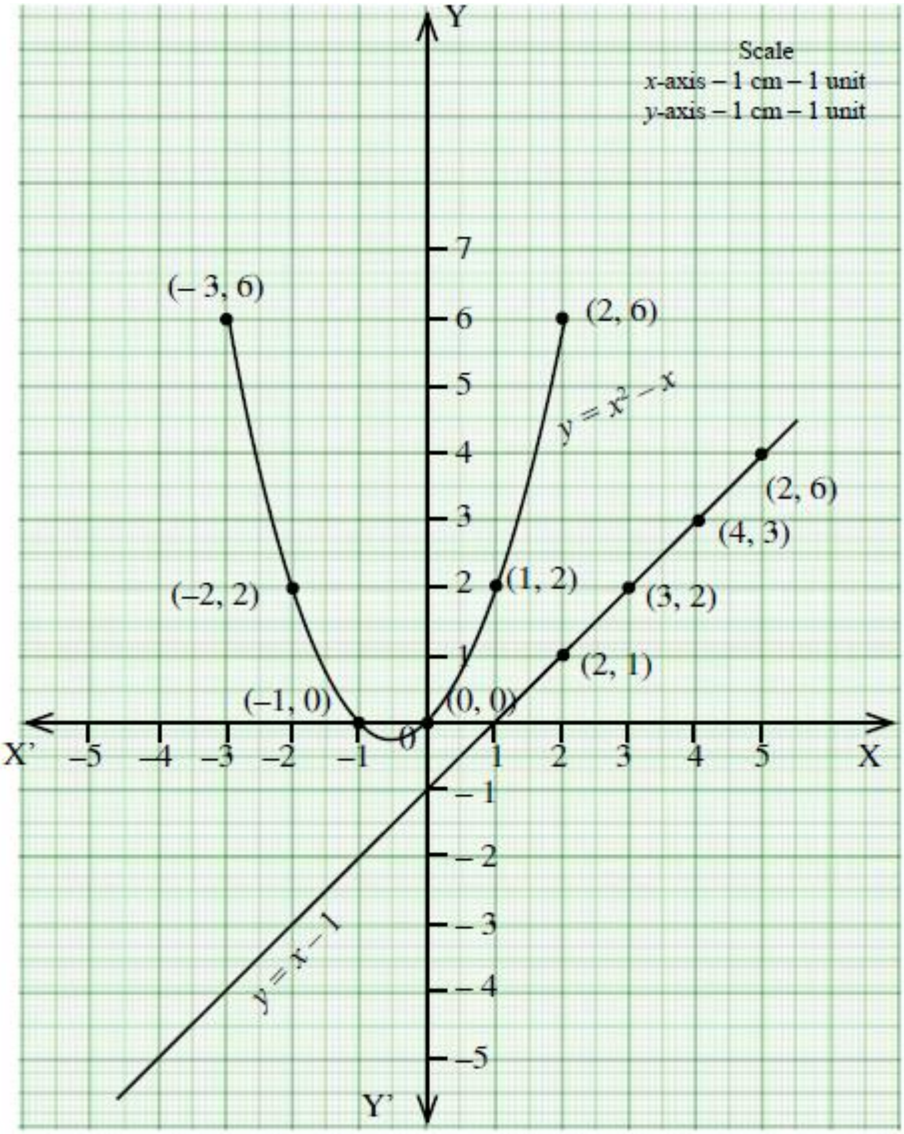
10. Draw the graph of $Y = X^2 + X$ and hence solve $X^2 + 1 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y	20	12	6	2	0	0	2	6	12	20	30

To solve $x^2 + 1 = 0$, subtract $x^2 + 1 = 0$ from $y = x^2 + x$.

$$\begin{array}{l} y = x^2 + \quad x \\ 0 = x^2 - 0x + 1 \\ \hline y = \qquad x - 1 \\ \hline \end{array}$$

x	– 4	– 3	– 2	– 1	0	1	2	3	4	5
y	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4



No Solution

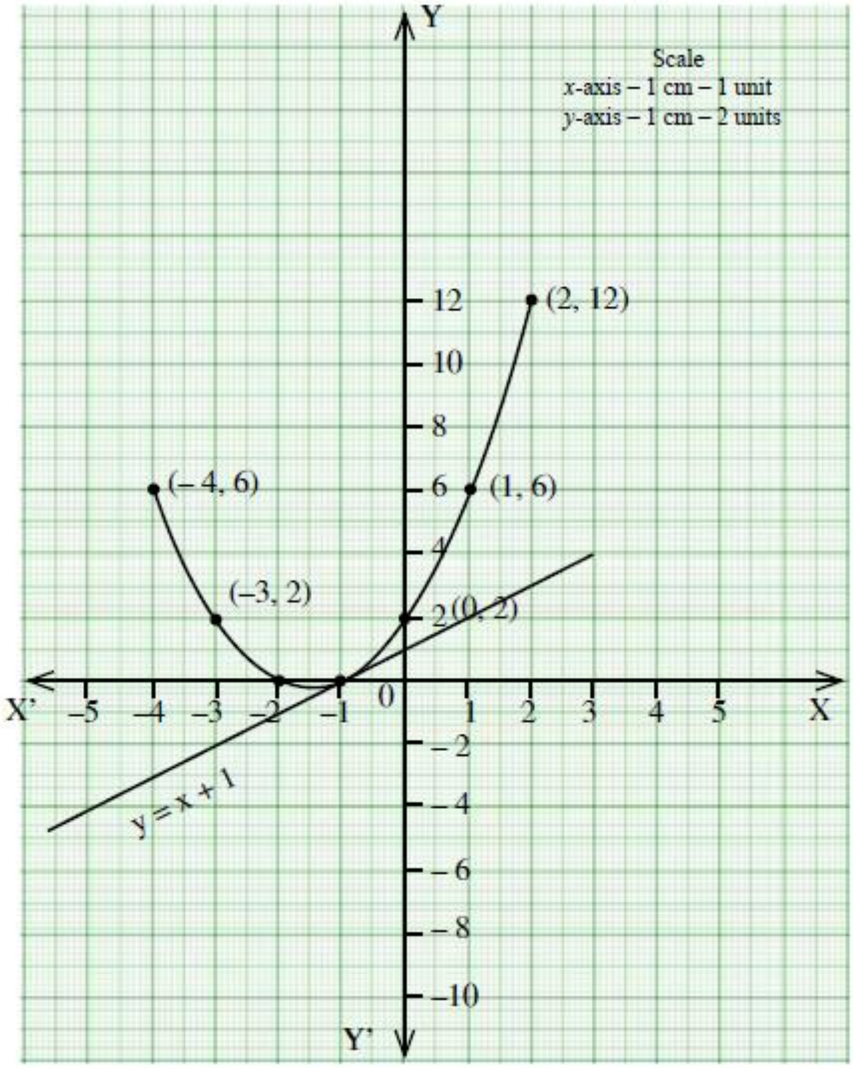
11. Draw the graph of Y = X^2 + 3x + 2 and use it to solve X^2 + 2x + 1 = 0

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
3X	-15	-12	-9	-6	-3	0	3	6	9	12	15
2	2	2	2	2	2	2	2	2	2	2	2
+ -	27 -15	18 -12	11 -9	6 -6	3 -3	2 0	6 0	12 0	20 0	30 0	42 0
Y	12	6	2	0	0	2	6	12	20	30	42

To solve $x^2 + 2x + 1 = 0$, subtract $x^2 + 2x + 1 = 0$ from $y = x^2 + 3x + 2$.

$$\begin{array}{l} y = x^2 + 3x + 2 \\ 0 = x^2 + 2x + 1 \\ \hline y = \qquad x + 1 \end{array}$$

x	– 4	– 3	– 2	– 1	0	1	2	3	4
y	– 3	– 2	– 1	0	1	2	3	4	5



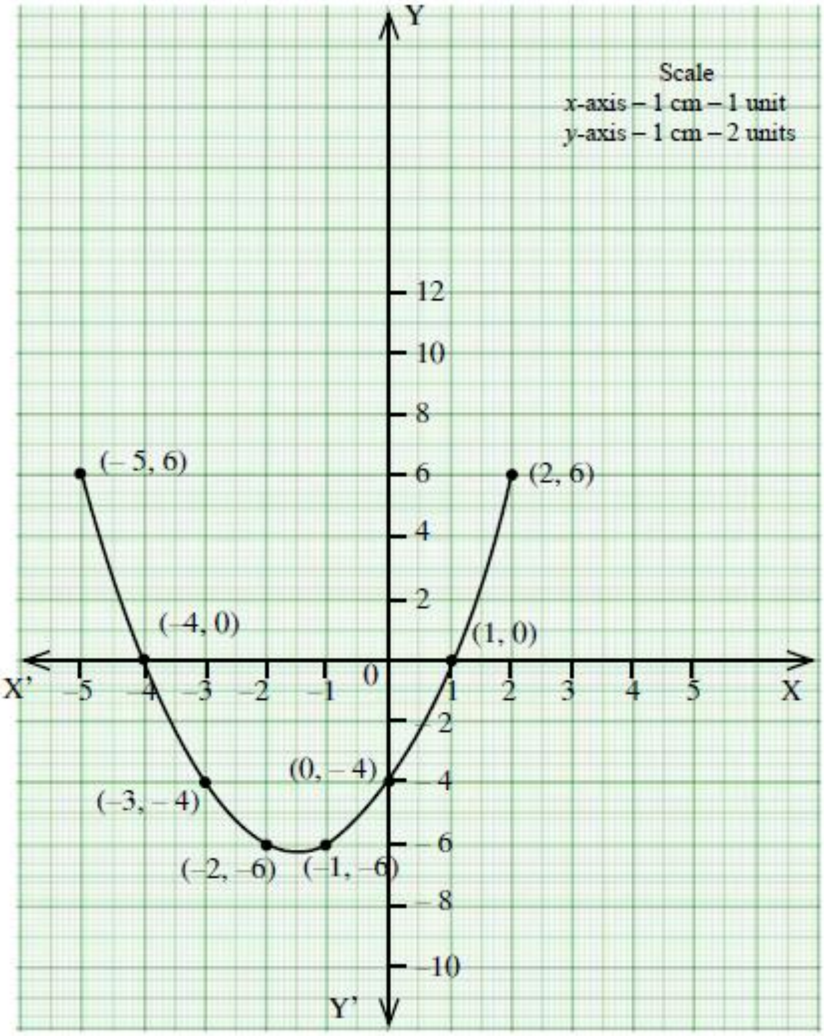
Solution : { -1, -1 }

12. Draw the graph of Y = X^2 + 3x – 4 and hence use it to solve x^2 + 3x – 4 – 0

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
3X	-15	-12	-9	-6	-3	0	3	6	9	12	15
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
+ -	25 -19	16 -16	9 -13	4 -10	1 -7	0 -4	4 -4	10 -4	18 -4	28 -4	30 -4
Y	6	0	-4	-6	-6	-4	0	6	14	24	26

To solve $x^2 + 3x - 4 = 0$, subtract $x^2 + 3x - 4 = 0$ from $y = x^2 + 3x - 4$.

$$\begin{array}{l} y = x^2 + 3x - 4 \\ 0 = x^2 + 3x - 4 \\ \hline y = \quad 0 \\ \hline \end{array}$$



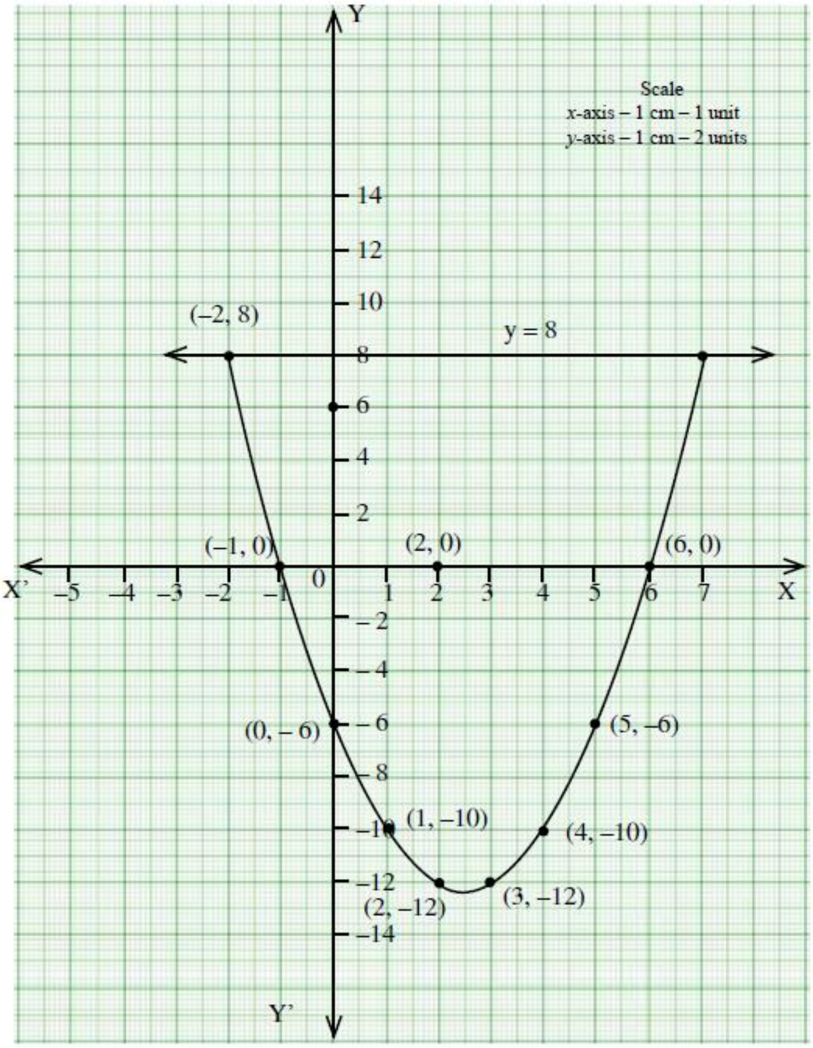
Solution : { -4, 1)

13. ***Draw the graph of*** $Y = X^2 - 5X - 6$ ***and hence solve*** $X^2 - 5X - 14 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
X^2	25	16	9	4	1	0	1	4	9	16	25	36	49
$-5X$	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
+ -	50 -6	36 -6	24 -6	14 -6	6 -6	0 -6	1 -11	4 -16	9 -21	16 -26	25 -31	36 -36	49 -41
Y	44	30	18	8	0	-6	-10	-12	-12	-10	-6	0	8

To solve $x^2 - 5x - 14 = 0$, subtract $x^2 - 5x - 14 = 0$ from $y = x^2 - 5x - 6$.

$$\begin{array}{l} y = x^2 - 5x - \quad 6 \\ 0 = x^2 - 5x - 14 \\ \hline y = \qquad 8 \\ \hline \end{array}$$



Solution : { -2, 7 }

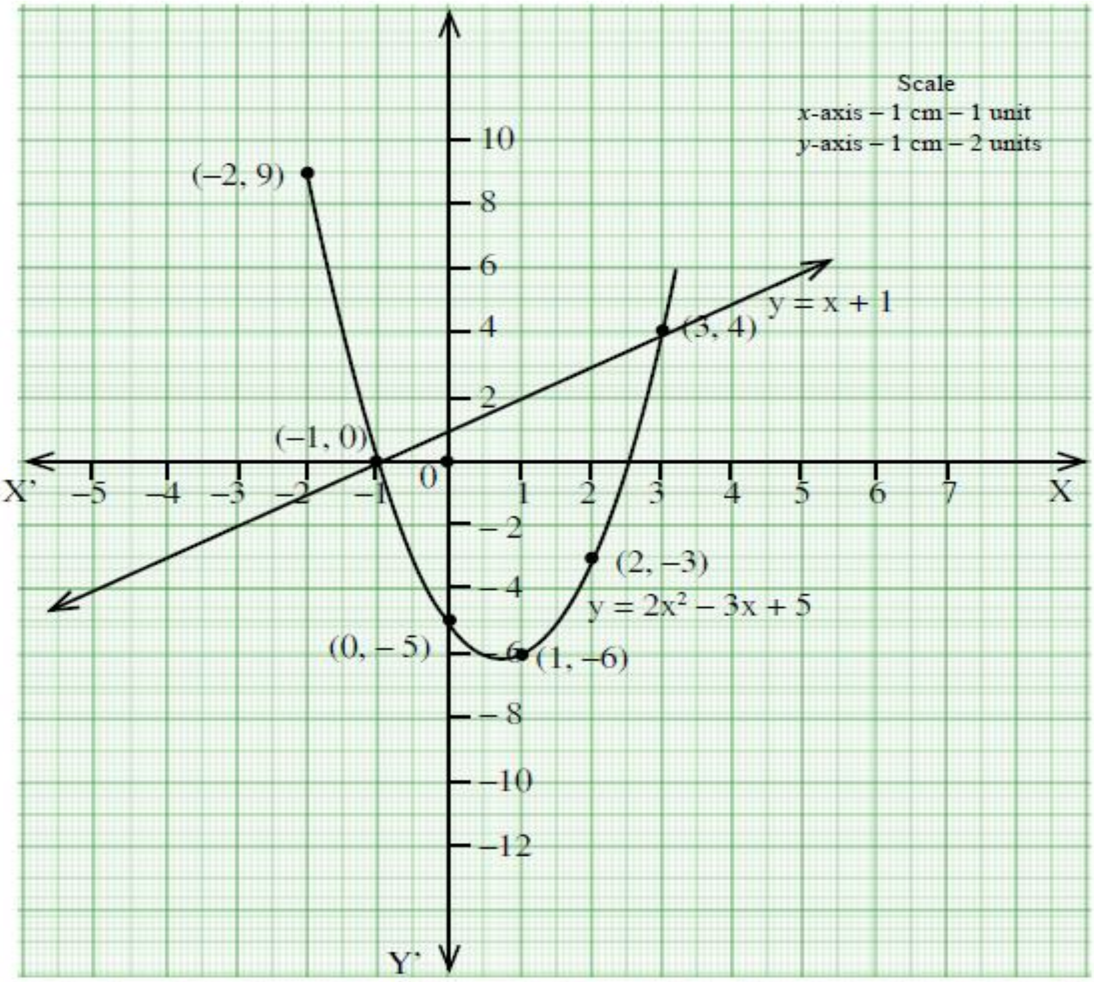
14. Draw the graph of $Y = 2x^2 – 3x – 5$ and hence solve $2x^2 – 4x – 6 = 0$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
$2X^2$	50	32	18	8	2	0	2	8	18	32	50
-3x	15	12	9	6	3	0	-3	-6	-9	-12	-15
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
+ -	65 -5	44 -5	27 -5	14 -5	5 -5	0 -5	2 -8	8 -11	18 -14	32 -17	50 -20
Y	60	39	22	9	0	-5	-6	-3	4	15	30

To solve $2x^2 – 4x – 6 = 0$, subtract it from $y = 2x^2 – 3x – 5$.

$$
\begin{aligned}
y &= 2x^2 – 3x – 5 \\
0 &= 2x^2 – 4x – 6 \\
\hline
y &= \qquad x + 1 \\
\hline
\end{aligned}
$$

X	0	1	2	-1
y	1	2	3	0



Solution : { -1, 3 }

15. ***Draw the graph of $Y = (X - 1)(X + 3)$ and hence solve $X^2 - X - 6 = 0$***

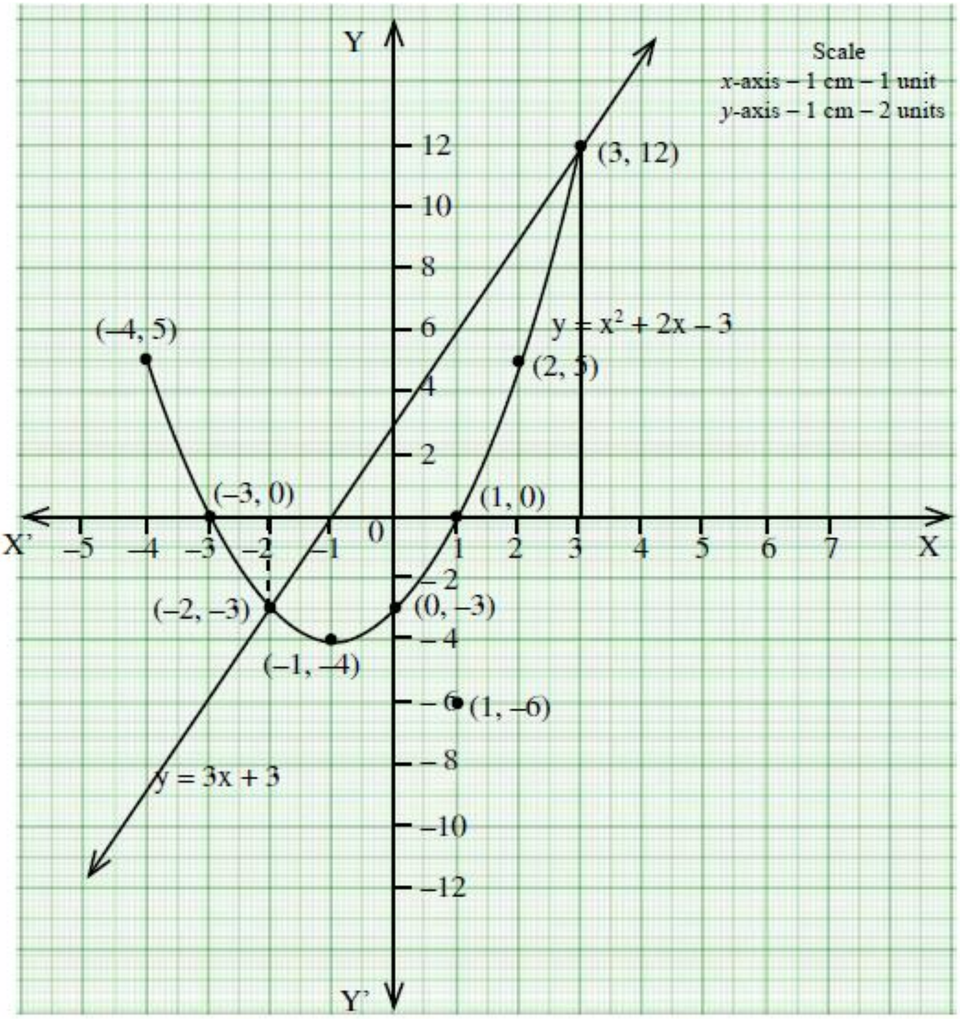
$Y = x^2 + 2x - 3$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X^2	25	16	9	4	1	0	1	4	9	16	25
2X	-10	-8	-6	-4	-2	0	2	4	6	8	10
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
+ -	25 -13	16 -11	9 -9	4 -7	1 -5	0 -3	3 -3	8 -3	15 -3	24 -3	35 -3
Y	12	5	0	-3	-4	-3	0	5	12	21	32

To solve $x^2 - x - 6 = 0$, subtract it from $y = x^2 + 2x - 3$.

$$\begin{array}{l} y = x^2 + 2x - 3 \\ 0 = x^2 - \ \ x - 6 \\ \hline y = \qquad 3x + 3 \\ \hline \end{array}$$

X	0	1	2	-1
y	3	6	9	0



Solution **:** ***{ -2, 3 }***

CHAPTER – 1

1. *If n (A x B) = 6 and A = (1, 3 } then n (B) is ……………….*
   a) 1 b) 2 c) 3 d) 6
2. *A = { a, b, p }, B { 2, 3 } and C = { p, q, r, s } then n [(A U C) x B] is ……………..*
   a) 8 b) 20 c) 12 d) 16
3. *If A = { 1, 2 }, B { 1, 2, 3, 4 }, C = { 5, 6 } and D = { 5, 6, 7, 8 } then state which of the following statement is true.*
   a) (A x C) $\subset$ (B x D) b) (B x D) $\subset$ (A x C) c) (A x B) $\subset$ (A x D) d) (D x A) $\subset$ (B x A)
4. *If there are 1024 relations from a set A = (1, 2, 3, 4, 5 } to a set B, then the number of elements in B is*
   a) 3 b) 2 c) 4 d) 8
5. *The range of the relation R =* $\{(x, x^2) \mid x \text{ is a prime number lessthan } 13\}$ *is*
   a) { 2, 3, 5, 7 } b) { 2, 3, 5, 7, 11 } c) { 4, 9, 25, 49, 121 } d) { 1, 4, 9, 25, 49, 121 }
6. *If the ordered pairs (a + 2, 4) and (5, 2a+b) are equal then (a, b) is*
   a) (2, -2) b) (5, 1) c) (2, 3) d) (3, -2)
7. *Let n(A) = m and n(B) = n then the total number of non – empty relations that can be defined from A to B is*
   a) m^n *b)* $n^{m,}$ *c)* $2^{mn} - 1$ *d)* 2^{mn}
8. *If { (a, 8), (6, b) } represents an identity function, then the value of a and b are respectively*
   a) (8, 6) b) (8, 8) c) (6, 8) d) (6, 6)
9. *Let A = { 1, 2, 3, 4 } and B { 4, 8, 9, 10 }. A function f : A $\rightarrow$ B given by*
   f = { (1, 4), (2, 8), (3, 9), (4, 10)} is a
   a) Many – one function b) Identity function
   c) One – to – one function d) Into function
10. *If f(x) = 2x² and g(x) =* $\frac{1}{3x}$ *then (f o g) is*
    a) $\frac{3}{2x^2}$ *b)* $\frac{2}{3x^2}$ *c)* $\frac{2}{9x^2}$ *d)* $\frac{1}{6x^2}$
11. *If f : A $\rightarrow$ B is a bijective function and if n(B) = 7, then n(A) is equal to*
    a) 7 b) 49 c) 1 d) 14
12. *Let f and g be two functions given by f { (0, 1), (2, 0), (3, -4), (4, 2), (5, 7) }*
    G = { (0, 2), (1, 0), (2, 4), (-4, 2), (7, 0) } then the range of (f o g) is
    `a) { 0, 2, 3, 4, 5 } b) { -4, 1, 0, 2, 7 } c) { 1, 2, 3, 4, 5 } d) { 0, 1, 2 }
13. *Let f(x) =* $\sqrt{1 + x^2}$ *then*
    a) f(xy) = f(x) . f(y) b) $f(xy) \geq f(x) . f(y)$ *c)* $f(xy) \leq f(x) . f(y)$ *d) None of these*
14. *If g { (1, 1), (2, 3), (3, 5), (4, 7) } is a function given by g(x) =* $\alpha(x) + \beta$ *then the values of* α *and* β *are*
    a) (-1, 2) b) (2, -1) c) (-1, -2) d) (1, 2)
15. $f(x) = (x + 1)^3 - (x - 1)^3$ *represents a function which is*
    a) linear b) cubic c) reciprocal d) quadratic

CHAPTER – 2

1. *Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy*
   a) 1 < r < b b) 0 < r < b c) $0 \leq r < b$ *d)* $0 < r \leq b$
2. *Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the remainders are*
   a) 0, 1, 8 b) 1, 4, 8 c) 0, 1, 3 d) 1, 3, 5

3. If the HCF of 65 and 117 is expressible in the form of 65m – 117, then the value of m is
a) 4 b) 2 c) 1 d) 3

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
a) 1 b) 2 c) 3 d) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
a) 2025 b) 5220 c) 5025 d) 2520

6. 7^{4k} = ________________(mod 100)
a) 1 b) 2 c) 3 d) 4

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is

a) 3 b) 5 c) 8 d) 11

8. The first of an arithmetic progoresosion is unity and the common difference is 4. Which of the following will be a term of this A. P.
a) 4551 b) 10091 c) 7881 d) 13531

9. If 6 items of 6th term of an A. P. is equal to 7 times the 7th term, then the 13th term of the A. P. is
a) 0 b) 6 c) 7 d) 13

10. An A. P. consists of 31 terms. If its 16th term is 'm', then the sum of all the terms of this A. P. is
a) 16m b) 62m c) 31m d) $\frac{31}{2}m$

11. In an A. P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?
a) 6 b) 7 c) 8 d) 9

12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \ldots\ldots + 2^0$ which of the following is true?
a) B is 2^{64} more than A b) A and B are equal
c) B is larger than A by 1 d) A is larger than B by 1

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots\ldots\ldots\ldots$ is
a) $\frac{1}{24}$ b) $\frac{1}{27}$ c) $\frac{2}{3}$ d) $\frac{1}{81}$

14. If the sequence t_1, t_2, t_3, …………are in A.P then the sequence t_6, t_{12}, t_{18}, ……………. is
a) a Geometric Progression b) an Arithmetic Progression
c) neither an Arithmetic Progression nor a Geometric Progression
d) a constant sequence

15. The value of $(1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots.15^3) - (1 + 2 + 3 + \ldots\ldots\ldots\ldots + 15)$ is
a) 14400 b) 14200 c) 14280 d) 14520

CHAPTER – 3

1. A system of three linear equations in three variables is inconsistent if their planes
a) intersect only at a point b) intersect in a line
a) coincides with each other d) do not intersect

2. The solution of the system x + y -3z = -6, -7y + 7z = 7, 3z = 9 is
a) x = 1, y = 2, z = 3 b) x = -1, y = 2, z = 3 c) x = -1, y = -2, z = 3
d) x = 1, y = 2, z = 3

3. If (x – 6) is the H. C. F of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
a) 3 b) 5 c) 6 d) 8

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$
a) $\frac{9y}{7}$ b) $\frac{9y^2}{(21y-21)}$ c) $\frac{21y^2-42y+21}{3y^2}$ d) $\frac{7(y^2-2y+1)}{y^2}$

5. $y^2 + \frac{1}{y^2}$ *is not equal to*

a) $\frac{y^4 + 1}{y^2}$ b) $\left(y + \frac{1}{y}\right)^2$ c) $\left(y - \frac{1}{y}\right)^2 + 2$ d) $\left(y + \frac{1}{y}\right)^2 - 2$

6. $\frac{x}{x^2 - 25} - \frac{8}{x^2 + 6x + 5}$

a) $\frac{x^2 - 7x + 40}{(x-5)(x+5)}$ b) $\frac{x^2 + 7x + 40}{(x-5)(x+5)(x+1)}$ c) $\frac{x^2 - 7x + 40}{(x^2 - 25)(x+1)}$ d) $\frac{x^2 + 10}{(x^2 - 25)(x+1)}$

7. *The square root of* $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ *is equal to*

a) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ b)16 $\left|\frac{y^2}{x^2z^4}\right|$ c) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ d) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

8. *Which of the following should be added to make* $x^4 + 64$ *a pefect square*

a) $4x^2$ *b)* $16x^2$ *c)* $8x^2$ *d)* $-8x^2$

9. *The solution of* $(2x - 1)^2 = 9$ *is equal to*

a) -1 b) 2 c) -1, 2 d) None of these

10. *The values of a and b if* $4x^4 - 24x^3 + 76x^2 + ab + b$ *is a perfect square are*

a) 100, 120 b) 10, 12 c) -120, 100 d) 12, 10

11. *If the roots of the equation* $q^2x^2 + p^2x + r^2 = 0$ *are the squares of the roots of the equation* $qx^2 + px + r = 0$*, then q, p, r are in*

a) A. P b) G.P c)Both A.P and G.P d) None of these

12. *Graph of a linear polynomial is a*

a) straight line b) circle c) parabola d) hyperbola

13. *The number of points of intersection of the quadratic polynomial* $x^2 + 4x + 4$ *with the X – axis is*

a) 0 b) 1 c) 0 or 1 d) 2

14. *For the given matrix A =* $\begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ *the order of the matri* A^T *is*

a) 2 x 3 b) 3 x 2 c) 3 x 4 d) 4 x 3

15. *If A is a 2 x 3 matrix and B is a 3 x 4 matrix, how many columns does AB have*

a) 3 b) 4 c) 2 d) 5

16. *If number of columns and rows are not equal in a matrix then it is said to be a*

a) diagonal matrix b) rectangular matrix c) square matrix d) identity matrix

17. *Transpose of a column matrix is*

a) unit matrix b) diagonal matrix c) column matrix d) row matrix

18. *Find the matrix X if 2X +* $\begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$

a) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. *Which of the following can be calculated from the given matrices*

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ *(i)* A^2 *(ii)* B^2 *(iii) AB (iv) BA*

a) (i) and (ii) only b) (ii) and (iii) only c) (ii) and (iv) only d) all of these

20. *If* $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ *and* $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$ *Which of the following statements are correct (i) AB + C =* $\begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ *(ii) BC =* $\begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$

(iii) BA + C = $\begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ *(iv) (AB)C =* $\begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

a) (i) and (ii) only b) (ii) and (iii) only c) (iii) and (iv) only d) all of these

CHAPTER – 4

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

a) $\angle B = \angle E$ b) $\angle A = \angle D$ c) $\angle B = \angle D$ d) $\angle A = \angle F$

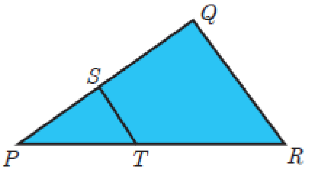
2. In Δ LMN, $\angle L = 60^0$, $\angle M = 50^0$, If Δ LMN $\sim \Delta$ PQR then the value of $\angle R$ is

a) 40^0 b) 70^0 c) 30^0 d) 110^0

3. If Δ ABC is an isosceles triangle with $\angle C = 90^0$ and AC = 5cm, then AB is

a) 2.5cm b) 5cm c) 10cm d) $5\sqrt{2}$ cm

4. In a given figure ST | | QR, PS = 2cm and SQ = 3cm. Then the ratio of the area of Δ PQR to the area of Δ PST is



a) 25 : 4 b) 25 : 7

c) 25 : 11 b) 25 : 13

5. The perimeters of two similar triangles Δ ABC and Δ PQR are 36cm and 24cm respectively. If PQ = 10c, then the length of AB is

a) $6\frac{2}{3}$ cm b) $\frac{10\sqrt{6}}{3}$ cm c) $66\frac{2}{3}$ cm d) 15cm

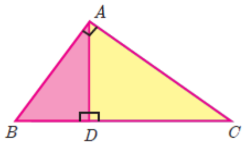
6. If in Δ ABC, DE | | BC, AB = 3.6cm, AC = 2.4cm and AD = 2.1cm then the length of AE is

a) 1.4 cm b) 1.8 cm c) 1.2 cm d) 1.05 cm

7. In a Δ ABC, AD is the bisector of $\angle$BAC. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is

a) 6 cm b) 4 cm c) 3 cm d) 8 cm

8. In the adjacent figure $\angle BAC = 90^0$ and AD $\perp$ BC then



a) BD . CD = BC^2 b) AB. AC = BC^2

c) BD.CD = AD^2 d) AB.AC = AD^2

9. Two poles of heights 6 m and 11 m stand vertically on the plane ground. If the distance between their feet is 12 m, what is the distance between their tops ?

a) 13 m b) 14 m c) 15 m d) 12.8 m

10. In the given figure, PR = 26 cm, QR = 24 cm, $\angle PAQ = 90^0$, PA = 6 cm and QA = 8 cm. Find $\angle$PQR

a) 80^0 b) 85^0

c) 75^0 d) 90^0

11. A tangent is perpendicular to the radius at the

a) centre b) point of contact c) infinity d) chord.

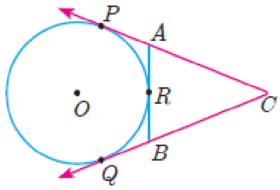
12. How many tangents can be drawn to the circle from an exterior point ?

a) one b) two c) infinite d) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^0$ then the value of $\angle$AOB is

a) 100^0 b) 110^0 c) 120^0 d) 130^0

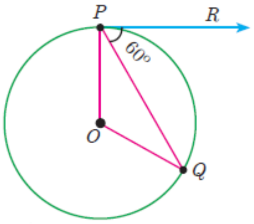
14. In the figure CP and CQ are tangents to a circle with centre at O. ARB is another touching the circle at R. If CP = 11cm and BC = 7cm, then the length of BR is



a) 6 cm b) 5 cm

c) 8 cm d) 4cm

15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then ∠POQ is
a) 120^0 b) 100^0
c) 110^0 d) 90^0



CHAPTER – 5

1. The area of triangle formed by the points (-5, 0), (0, -5) and (5, 0) is
a) 0 sq. units b) 25 sq. units c) 5 sq. units d) none of these
2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is
a) x = 10 b) y = 10 c) x = 0 d) y = 0
3. The straight line given by the equation x = 11 is
a) parallel to X axis b) parallel to Y axis
c) passing through the origin d) passing through the point (0, 11)
4. If (5, 7) , (3, p) and (6, 6) are collinear, then the value of p is
a) 3 b) 6 c) 9 d) 12
5. The point of intersection of 3x – y = 4 and x + y = 8 is
a) (5, 3) b) (2, 4) c) (3, 5) d) (4, 4)
6. The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is
a) 1 b) 4 c) -5 d) 2
7. The slope of the line which is perpendicular to line joining the points (0, 0) and (-8, 8) is
a) -1 b) 1 c) $\frac{1}{3}$ d) -8
8. The slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of perpendicular bisector of PQ is
a) $\sqrt{3}$ b) - $\sqrt{3}$ c) $\frac{1}{\sqrt{3}}$ d) 0
9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is
a) 8x + 5y = 40 b) 8x – 5y = 40 c) x = 8 d) y = 5
10. The equation of a line passing through the origin and perpendiuclar to the line 7x – 3y + 4 = 0 is
a) 7x – 3y + 4 = 0 b) 3x – 7y + 4 = 0 c) 3x + 7y = 0 d) 7x – 3y = 0
11. Consider four straight lines
(i) $l_1 : 3y = 4x + 5$ (ii) $l_2 : 4y = 3x - 1$ (iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$
Which of the following statement is true ?
a) l_1 & l_2 are perpendicular b) l_1 & l_4 are parallel
c) l_2 & l_4 are perpendicular d) l_2 & l_3 are parallel
12. A straight line has equation 8y = 4x + 21. Which of the following is true
a) The slope is 0.5 and the y intercept is 2.6 b) The slope is 5 and the y intercept is 1.6
c) The slope is 0.5 and the y intercept is 1.6 d) The slope is 5 and the y intercept is 2.6
13. When proving that a quadrilateral is trapezium, it is necessary to show
a) Two sidies are parallel b) Two parallel and two non parallel sides
c) Opposite sides are parallel d) all sides are of equal length
14. When proving that a quadrilateral is a parallelogram by using slopes you must find

a) The slopes of four sides b) The slopes of two pair of opposite sides
c) The lengths of all sides d) Both the lengths and slopes of two sides

15. (2, 1) is the point of intersection of two lines
a) $x - y - 3 = 0$; $3x - y - 7 = 0$ b) $x + y = 3$; $3x + y = 7$
c) $3x + y = 3$; $x + y = 7$ d) $x + 3y - 3 = 0$; $x - y - 7 = 0$

CHAPTER – 6

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to
a) $\tan^2\theta$ b) 1 c) $\cot^2\theta$ d) 0

2. $\tan\theta\,\text{cosec}^2\theta - \tan\theta$ is equal to
a) $\sec\theta$ b) $\cot^2\theta$ c) $\sin\theta$ d) $\cot\theta$

3. If $(\sin\alpha + \text{cosec}\,\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to
a) 9 b) 7 c) 5 d) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \text{cosec}\,\theta = b$, then the value of $b(a^2 - 1)$ is equal to
a) 2a b) 3a c) 0 d) 2ab

5. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to
a) 25 b) $\frac{1}{25}$ c) 5 d) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to
a) $\frac{-3}{2}$ b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then
a) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ c) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ is equal to
a) 0 b) 1 c) 2 d) -1

9. $a\cot\theta + b\,\text{cosec}\,\theta = p$ and $b\cot\theta + a\,\text{cosec}\,\theta = q$ then $p^2 - q^2$ is equal to
a) $a^2 - b^2$ b) $b^2 - a^2$ c) $a^2 + b^2$ d) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$, then the angle of elevation of the sun has measure
a) 45^0 b) 30^0 c) 90^0 d) 60^0

11. The electric pole subtends an angle of 30^0 at a point on the same level as its foot. At a second point'b' meters above the first, the depression of the foot of the tower is 60^0. The height of the tower (in meters) is equal to
a) $\sqrt{3}b$ b) $\frac{b}{3}$ c) $\frac{b}{2}$ d) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m height. Its shadow is x meters shorter when the sun's altitude is 45^0 then when it has been 30^0, then 'x' is equal to
a) 41.92 m b) 43.92 m c) 43m d) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30^0 and 60^0 respectively. The height of the multistoried building and distance between two buildings (in meters) is
a) 20, $10\sqrt{3}$ b) 30, $5\sqrt{3}$ c) 20, 10 d) 30, $10\sqrt{3}$

14. Two persons are standing 'x' meters apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in meters) is

a) $\sqrt{2}x$ b) $\frac{x}{2\sqrt{2}}$ c) $\frac{x}{\sqrt{2}}$ d) 2x

15. The angle of elevation of a cloud from a point h meters above a lake is β. The angle of depression of its reflection in the lake is 45^0. The height of location of the cloud from the lake is

a) $\frac{h(1+tan\beta)}{1-tan\beta}$ b) $\frac{h(1-tan\beta)}{1+tan\beta}$ c) h tan $(45^0 - \beta)$ d) none of these

CHAPTER – 7

1. The curved surface area of a right circular cone of height 15cm and base diameter 16 cm is

a) $60\pi cm^2$ b) $68\pi cm^2$ c) $120 \pi cm^2$ d) $136 \pi cm^2$

2. If two solid hemispheres of same base radius 'r' units are joined together along their bases, then curved surface area of this new solid is

a) $4\pi r^2$ sq. units b) $6\pi r^2$ sq. units c) $3\pi r^2$ sq. units d) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5cm and slant height is 13cm will be

a) 12 cm b) 10 cm c) 13 cm d) 5 cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

a) 1 : 2 b) 1 : 4 c) 1 : 6 d) 1 : 8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

a) $\frac{9\pi h^2}{8}$ sq. units b) $24\pi h^2$ sq. units c) $\frac{8\pi h^2}{9}$ sq. units d) $\frac{56\pi h^2}{9}$ sq. units

6. In a hollow cylinder, the sum of the external and internal radii is 14cm and the width is 4cm. If its height is 20 cm, the volume of the material in it is.

a) $5600\pi cm^3$ b) $11200\pi cm^3$ c) $56\pi cm^3$ d) $3600\pi cm^3$

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is

a) made 6 times b) made 18 times c) made 12 times d) unchanged

8. The total surface area of a hemi – sphere is how much times the square of its radius

a) π b) 4π c) 3π d) 2π

9. A solid sphere of radius 'x' cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

a) 3x cm b) x cm c) 4x cm d) 2x cm

10. A frustum of a right circular cone is of height 16cm with radii of its ends as 8cm and 20cm. Then, the volume of the frustum is

a) $3328\pi cm^3$ b) $3228\pi cm^3$ c) $3240\pi cm^3$ d) $3340\pi cm^3$

11. A shuttle cock used for playing badminton has the shape of the combination of

a) a cylinder and a sphere b) a hemisphere and a cone

c) a sphere and a cone d) frustum of a cone and a hemisphere

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

a) 2 : 1 b) 1 : 2 c) 4 : 1 d) 1 : 4

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5cm is

a) $\frac{4}{3}\pi$ b) $\frac{10}{3}\pi$ c) 5π d) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is part are h_1 units and r_1 units respectively. Height of the frustum is h_2 and radius of the smaller base r_2 units.
If $h_1 : h_2 = 1: 2$ then $r_1 : r_2$ is

a) 1 : 3 b) 1 : 2 c) 2 : 1 d) 3 : 1

15. *The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is*
a) 1 : 2 : 3 b) 2 : 1 : 3 c) 1 : 3 : 2 d) 3 : 1 : 2

CHAPTER – 8

1. *Which of the following is not a measure of dispersion?*
a) Range b) Standard deviation c) Arithmetic mean d) Variance
2. *The range of the data 8,8,8,8,8,……………8 is*
a) 0 b) 1 c) 8 d) 3
3. *The sum of all deviations of the data from its mean is*
a) Always positive b) always negative c) zero d) non – zero integer
4. *The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is*
a) 40000 b) 160900 c) 160000 d) 30000
5. *Variance of first 20 natural numbers is*
a) 32.25 b) 44.25 c) 33.25 d) 30
6. *The standard deviation of a data is 3. If each value is multipled by 5 then the new variance is*
a) 3 b) 15 c) 5 d) 225
7. *If the standard deviation of x, y, z is 'p' then the standard deviation 3x + 5, 3y + 5, 3z + 5 is*
a) 3p + 5 b) 3p c) p + 5 d) 9p + 15
8. *If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is*
a) 3.5 b) 3 c) 4.5 d) 2.5
9. *Which of the following is incorrect ?*
a) P(A) > 1 b) 0 ≤ P(A) ≤ 1 c) P(ϕ) = 0 d) P(A) + P($\bar{A}$) = 1
10. *The probability a red marble selected at random from a jar conotaining 'p' red, 'q' blue and 'r' green marbles is*
a) $\frac{q}{p+q+r}$ *b)* $\frac{p}{p+q+r}$ *c)* $\frac{p+q}{p+q+r}$ *d)* $\frac{p+r}{p+q+r}$
11. *A page is slected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is*
a) $\frac{3}{10}$ *b)*$\frac{7}{10}$ *c)*$\frac{3}{9}$ *d)*$\frac{7}{9}$
12. *The probability of getting a job for a person is* $\frac{x}{3}$. *If the probability of not getting the job is* $\frac{2}{3}$ *then the value of 'x' is*
a) 2 b) 1 c) 3 d) 1.5
13. *Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is* $\frac{1}{9}$*, then the number of tickets bought by Kamalam is*
a) 5 b) 10 c) 15 d) 20
14. *If a letter is chosen at random from the English alphabets { a, b, c, …….., z}, then the probability that the letter chose precedes 'x'*
a) $\frac{12}{13}$ *b)*$\frac{1}{13}$ *c)*$\frac{23}{26}$ *d)*$\frac{3}{26}$
15. *A purse contains 10 notes of ₹2000, 15 notes ₹5oo and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note ?*
a)$\frac{1}{5}$ *b)*$\frac{3}{10}$ *c)*$\frac{2}{3}$ *d)*$\frac{4}{5}$

ANSWERS

CHAPTER – 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	c	a	b	c	a	c	a	c	c	a	d	c	b	d

CHAPTER – 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	a	b	c	d	a	d	c	a	c	c	d	b	b	c

CHAPTER – 3

1	2	3	4	5	6	7	8	9	10
d	a	b	a	b	c	d	b	c	c
11	12	13	14	15	16	17	18	19	20
b	a	b	c	b	b	d	b	b	a

CHAPTER – 4

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	d	a	d	a	b	c	a	d	b	b	b	d	a

CHAPTER – 5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	b	c	c	d	b	b	a	c	c	a	b	a	b

CHAPTER – 6

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	d	b	a	b	b	a	c	b	d	b	b	d	b	a

CHAPTER – 7

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	a	a	b	c	b	b	c	c	a	d	a	a	b	d

CHAPTER –8

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	a	c	b	c	d	b	a	a	b	b	b	c	c	d

CHAPTER – 1

1. If $A = \{ 1, 3, 5 \}$ and $B = \{ 2, 3 \}$ then find (i) $A \times B$ and (ii) $B \times A$

Answer:

Given that $A = \{ 1, 3, 5 \}$ and $B = \{ 2, 3 \}$

(i) $A \times B = \{ 1, 3, 5 \} \times \{ 2, 3 \} = \{ (1,2),(1,3),(3,2),(3,3),(5,2),(5,3), \}$

(ii) $B \times A = \{ 2, 3 \} \times (1, 3, 5 \} = \{ (2,1),(2,3),(2,5),(3,1),(3,3),(3,5), \}$

2. If $A \times B = \{ (3, 2), (3, 4), (5, 2), (5, 4) \}$ then find A and B.

Answer:

We have A = { set of all first coordinates of the elements of A x B } = { 3, 5 }

B = { Set of all second coordinates of the elements of A x B } = { 2, 4 }

3. Let $A = \{ x \in N \mid 1 < x < 4 \}$, $B = \{ x \in W \mid 0 \leq x < 2 \}$ and $C = \{ x \in N \mid x < 3 \}$ then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Answer:

Given : $A = \{ 2, 3 \}$, $B = \{ 0, 1 \}$ and $C = \{ 1, 2 \}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS : $A \times (B \cup C)$

$(B \cup C) = \{ 0, 1 \} \cup \{ 1, 2 \} = \{ 0, 1, 2 \}$

$A \times (B \cup C) = \{ 2, 3 \} \times \{ 0,1, 2 \} = \{ (2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ ------------- (1)

RHS: $(A \times B) \cup (A \times C)$

$(A \times B) = \{ 2, 3 \} \times \{ 0, 1 \} = \{ (2,0),(2,1),(3,0),(3,1)\}$

$(A \times C) = \{ 2, 3) \times \{ 1, 2 \} =\{ (2,1),(2,2),(3,1),(3,2) \}$

$(A \times B) \cup (A \times C) = \{ (2,0),(2,1),(3,0),(3,1)\} \cup \{ (2,1),(2,2),(3,1),(3,2) \}$

$\{ (2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ --------------(2)

From (1) and (2) **LHS = RHS** that is

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS : $A \times (B \cap C)$

$(B \cap C) = \{ 0,1\} \cap \{ 1, 2 \} = \{ 1 \}$

$A \times (B \cap C) = \{ 2, 3 \} \times \{ 1 \} = \{ (2,1),(3,1) \}$ ------------------- (1)

RHS: $(A \times B) \cap (A \times C)$

$(A \times B) = \{ 2, 3 \} \times \{ 0, 1 \} = \{ (2,0),(2,1),(3,0),(3,1)\}$

$(A \times C) = \{ 2, 3) \times \{ 1, 2 \} =\{ (2,1),(2,2),(3,1),(3,2) \}$

$(A \times B) \cap (A \times C) = \{ (2,0),(2,1),(3,0),(3,1)\} \times \{ (2,1),(2,2),(3,1),(3,2) \}$

$= \{ (2,1),(3,1)\}$ ---------------- (2)

From (1) and (2) **LHS = RHS** that is

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

4. Find A x B, A x A, B x A and B x B

(i) $A = \{ 2, -2, 3 \}$ and $B \{ 1, -4 \}$ (ii) $A = B = \{ p, q \}$ (iii) $A = \{ m, n \}$ and $B = \phi$

Answer:

(i) If $A = \{ 2, -2, 3)$ and $B (1, -4)$

$A \times B = \{ 2, -2, 3 \} \times \{ 1, -4 \} = \{ (2,1),(2,-4),(-2,1),(-2,-4),(3,1),(3,-4) \}$

$A \times A = \{ 2, -2, 3 \} \times \{ 2, -2, 3 \} = \{ (2,2),(2,-2),(2,3),(-2,2),(-2,-2),(-2,3),(3,2),$
$(3,-2),(3,3) \}$

$B \times A = \{1, -4\} \times \{2, -2, 3\}$ $= \{(1,2),(1,-2),(1,3),(-4,2),(-4,-2),(-4,3\}$

$B \times B = \{1, -4\} \times \{1, -4\}$ $= \{(1,1),(1,-4),(-4,1),(-4,-4)$

(ii) If $A = \{p, q\}$ and $B = \{p, q\}$

$A \times B = \{p, q\} \times \{p, q\}$ $= \{(p, p),(p, q),(q, p),(q, q)\}$

$A \times A = \{p, q\} \times \{p, q\}$ $= \{(p, p),(p, q),(q, p),(q, q)\}$

$B \times A = \{p, q\} \times \{p, q\}$ $= \{(p, p),(p, q),(q, p),(q, q)\}$

$B \times B = \{p, q\} \times \{p, q\}$ $= \{(p, p),(p, q),(q, p),(q, q)\}$

(iii) $A = \{m, n\}$ and $B = \{\}$ or ϕ

$A \times B = \{m, n\} \times \{\}$ $= \{\ \}$ or ϕ

$A \times A = \{m, n\} \times \{m, n\}$ $= \{(m, m),(m, n),(n, m),(n, n)\}$

$B \times A = \{\ \} \times \{m, n\}$ $= \{\}$ or ϕ

$B \times B = \{\ \} \times \{\ \}$ $= \{\ \}$ or ϕ

5. Let $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a prime number less than 10 $\}$. Find $A \times B$ and $B \times A$

Answer:

Given $A = \{1, 2, 3\}$ and $B = \{2, 3, 5, 7\}$

(i) $A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$ $=\{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7)\}$

(ii) $B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$ $=\{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),(5,2),(5,3),(7,1),(7,2),(6,3)\}$

6. If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3,3), (3, 4)\}$ find A and B

Answer:

Given : $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3,3), (3, 4)\}$

$A = \{3, 4\}$ and $B = \{-2, 0, 3\}$

7. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$. Show that $A \times A = (B \times B) \cap (C \times C)$.

Answer:

Given : $A = \{5, 6\}$, $B = \{4, 5, 6\}$ and $C = \{5, 6, 7\}$

RHS : $\mathbf{A \times A} = \{5, 6\} \times \{5, 6\} = \{(5, 5),(5, 6),(6, 5),(6, 6)\}$ ------------------ (1)

LHS : $(\mathbf{B \times B}) \cap (\mathbf{C \times C})$

$(B \times B) = \{4, 5, 6\} \times \{4, 5, 6\} = \{(4,4),(4,5),(4,6),(5,4),(5,5),(5,6),(6,4),(6,5),(6,6)\}$

$(C \times C) = \{5, 6, 7\} \times \{5, 6, 7\} = \{(5,5),(5,6),(5,7),(6,5),(6,6),(6,7),(7,5),(7,6),(7,7)\}$

$(B \times B) \cap (C \times C)$ $= \{(5, 5),(5, 6),(6, 5),(6, 6)\}$ ----------------------- (2)

From (1) and (2) LHS = RHS that is

$\mathbf{A \times A = (B \times B) \cap (C \times C)}$.

8. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D\{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Answer:

Given : $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D\{1, 3, 5\}$,

LHS = $\mathbf{(A \cap C) \times (B \cap D)}$

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$ $= \{3\}$ $(B \cap D\} = \{2, 3, 5\} \cap \{1, 3, 5\}$ $= \{3, 5\}$

$(A \cap C) \times (B \cap D)$ $= \{3\} \times \{3, 5\}$ $= \{(3, 3), (3, 5)\}$ ------------------ (1)

RHS= $\mathbf{(A \times B) \cap (C \times D)}$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$ $= \{(1,2),(1,3),(1,5),(2,2),(2,3),(2,5),(3,2),(3,3),(3,5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$ $= \{(3,1),(3,3),(3,5),(4,1),(4,3),(4,5)\}$

$(A \times B) \cap (C \times D)$ $= \{(3, 3), (3, 5)\}$ ---------------------- (2)

From (1) and (2) LHS = RHS that is

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

9. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Answer:

Given : $A = \{0, 1\}$, $B = \{2, 3, 4\}$ and $C = \{3, 5\}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS: $A \times (B \cup C)$

$(B \cup C) = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\} = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$--------------- (1)

RHS: $(A \times B) \cup (A \times C)$

$(A \times B) = \{0, 1\} \times \{2, 3, 4\} = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$(A \times C) = \{0, 1\} \times \{3, 5\} = \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cup (A \times C)$ = $\{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$--------------- (2)

From (1) and (2) LHS = RHS that is

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS : $A \times (B \cap C)$

$(B \cap C) = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$A \times (B \cap C) = \{0,1\} \times \{3\} = \{(0, 3),(1, 3)\}$ --------------------------(1)

RHS : $(A \times B) \cap (A \times C)$

$(A \times B) = \{0, 1\} \times \{2, 3, 4\} = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$(A \times C) = \{0, 1\} \times \{3, 5\} = \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ---------------------------(2)

From (1) and (2) LHS = RHS that is

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS: $(A \cup B) \times C$

$(A \cup B) = \{0,1\} \cup \{2, 3, 4\} = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C$ $= \{0, 1, 2, 3, 4\} \times \{3, 5\} = \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),$
$(4,5)\}$------------------ (1)

RHS : $(A \times C) \cup (B \times C)$

$(A \times C) = \{0, 1\} \times \{3, 5\} = \{(0,3),(0,5),(1,3),(1,5)\}$

$(B \times C) = \{2, 3, 4\} \times \{3, 5\} = \{(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$

$(A \times C) \cup (B \times C)$ = $\{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$--------------(2)

From (1) and (2) LHS = RHS that is

$(A \cup B) \times C = (A \times C) \cup (B \times C)$

10. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8 and C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **(ii)** $A \times (B - C) = (A \times B) - (A \times C)$

Answer:

Given : $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 5, 7\}$ and $C = \{2,\}$

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS : $(A \cap B) \times C$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\} = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\} = \{(2,2),(3,2),(5,2),(7,2)\}$ --------------- (1)

RHS : $(A \times C) \cap (B \times C)$

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2,\} = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\} = \{(2,2),(3,2),(5,2),(7,2)\}$

$(A \times C) \cap (B \times C) = \{(2,2),(3,2),(5,2),(7,2)\}$ --------------- (2)

From (1) and (2) *LHS = RHS that is*

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS : $A \times (B - C)$

$(B - C) = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),(7,3),(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),$
$(7,5),(1,7),(2,7),(3,7),(4,7),(5,7),(6,7),(7,7)\}$ --------------------------(1)

RHS : $(A \times B) - (A \times C)$

$(A \times B) = \{\{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$=\{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),$
$(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),(7,2),(7,3),(7,5),(7,7)\}$

$(A \times C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C) = \{(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),(7,3),(1,5),(2,5),(3,5),(4,5),$
$(5,5),(6,5),(7,5),(1,7),(2,7),(3,7),(4,7),(5,7),(6,7),(7,7)\}$ --------------------------(2)

From (1) and (2) *LHS = RHS that is*

$A \times (B - C) = (A \times B) - (A \times C)$

11. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f : A \rightarrow B$ be a function give by $f(x) = 3x - 1$. Resresent this function

(i) by arrow diagram **(ii) in a table form**

(iii) as a set of ordered pairs **(iv) in a garaphical form**

Answer:

Given: $A = \{1, 2, 3, 4\}$, $B = \{2, 5, 8, 11, 14\}$ and $f(x) = 3x - 1$

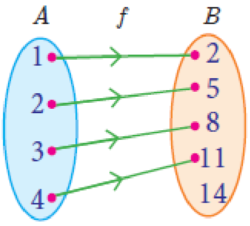
$f(1) = 3(1) - 1 = 3 - 1 = 2$

$f(2) = 3(2) - 1 = 6 - 1 = 5$

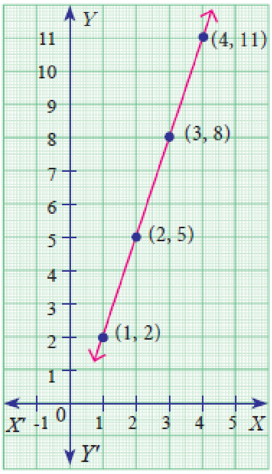
$f(3) = 3(3) - 1 = 9 - 1 = 8$

$f(4) = 3(4) - 1 = 12 - 1 = 11$

(i) An arrow diagram



(iv)graphical form



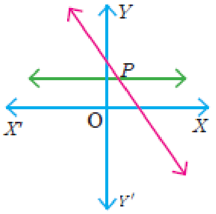
(ii) a table

x	1	2	3	4
f(x)	2	5	8	11

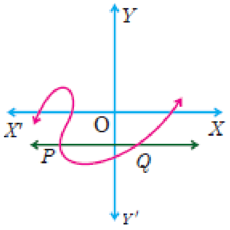
(iii) Set of ordered pairs $f = \{ (1, 2), (2, 5), (3, 8), (4, 11)\}$

12. ***Using horizontal line test the following given figures, determine which of the following functions are one – one.***

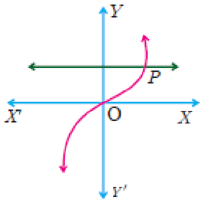
(i)



(ii)



(iii)



Answer:

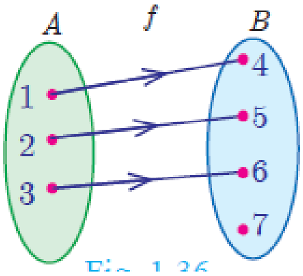
(i) *The curves in figure (i) represent a one – one function as the horizonotal lines meet the curves in only one point P.*

(ii) *The curve in figure (ii) does not represent a one – one function, since the horizontal line meet the curve in two points P and Q*

(iii) *The curves in figure (iii) represent a one – one function as the horizonotal lines meet the curves in only one point P.*

13. ***Let A = { 1, 2, 3 }, B = { 4, 5, 6, 7 } and f = { (1, 4). (2, 5), (3, 6) } be a function from A to B. Show that f is one – one but not onto function***

Answer:



Given : A = { 1, 2, 3 }, B = { 4, 5, 6, 7 } and f = { (1, 4), (2, 5), (3, 6)}

Then f is a function from A to B and for different elements in A, there are different images in B. Hence f is one – one function Note that the element 7 in the co – domain does not have any pre – image in the domain. Hence f is not onto.

There fore f is one – one but not an onto function. (or)

f is one – one function & into function

14. ***If A = { -2, -1, 0, 1, 2, } and f : A → B is an onto function defined buy*** $f(x) = x^2 + x + 1$ ***then find B.***

Answer:

Given: $A = \{ -2, -1, 0, 1, 2 \}$ and $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1 \quad = 4 - 2 + 1 \quad = 3$

$f(-1) = (-1)^2 + (-1) + 1 \quad = 1 - 1 + 1 \quad = 1$

$f(0) = (0)^2 + (0) + 1 \quad = 0 + 0 + 1 \quad = 1$

$f(1) = (1)^2 + (1) + 1 \quad = 1 + 1 + 1 \quad = 3$

$f(2) = (2)^2 + (2) + 1 \quad = 4 + 2 + 1 \quad = 7$

Since, f is an onto function, range of f = B = co – domain of f. *Therefore, B = { 1, 3, 7 }*

15. Let f be function $f : N \rightarrow N$ be defined by f(x) 3x + 2, $x \in N$

(i) Find the images of 1, 2, 3 (ii) Find the pre – images of 29, 53

(iii) Identify the type of function.

Answer: The function $f : N \rightarrow N$ be defined by f(x) 3x + 2, $x \in N$

(i) If x = 1 f(1) = 3(1) + 2 =3 + 2 = 5

If x = 2 f(2) = 3(2) + 2 =6 + 2 = 8

If x = 3 f(3) = 3(3) + 2 = 9 + 2 = 11

The images of 1, 2, 3, are 5, 8, 11 respectively.

(ii) If x is the pre – image of 29, then f(x) = 29. Hence 3x + 2 = 29

3x = 27 x = 27/9 **Therefore x = 9**

Similarly, if x is the preimage of 53, then f(x) = 53. Hence 3x + 2 = 53

3x = 51 x = 51/3 **Therefore x = 17**

(iii) Since different elements of N have different images in the co – domain, the function f is one – one function.

The co – domain of f is N

But the range of f = { 5, 8, 11, 14, 17 ……..} is a proper subset of N.

Therefore f is not an onto function. That is, f is an into function.

Thus f is one – one and into function.

16. Forensic scientists can determine the height (in cms) of a person based on the length of their thigh bone. They usually do so using the function h(b) = 2.47b + 54. 10 where 'b' is the length of the thigh bone.

(i) Check if the function 'h' is one – one

(ii) Also find the hight of a person if the length of his thigh bone is 50cms.

(iii) Find the length of the thigh bone if the height of a person is 147.96 cms.

Answer:

(i) To check if h is one – one, we assume that $h(b_1) = h(b_2)$.

Then we get, $2.47b_1 + 54.10 = 2.47b_2 + 54.10$

$2.47b_1 = 2.47b_2$

$b_1 = b_2$

Thus, $h(b_1) = h(b_2) \Rightarrow b_1 = b_2$. So the function h is one – one.

(ii) If the length of the thigh bone b = 50, then the height is

h(50) = (2.47 x 50) + 54. 10 = 112.50 + 54.10 = 177.6 cms.

(iii) If the heght of a person is 147.96cms, then h(b) = 147.96 and so the length of the thight bone is given by

2.47b + 54. 10 = 147.96

$$b = \frac{93.86}{2.47} = \frac{93.86 \times 100}{2.47 \times 100} = \frac{9386}{247} = 38$$

17. Let f be a function from R to R defined by f(x) = 3x – 5. Find the values of 'a' and 'b' given that (a, 4) and (1, b) belong to f.

Answer:

f(x) = 3x – 5 can be written as $f = \{ (x, 3x - 5) \mid x \in R\}$

(a, 4) means the image of a is 4. That is f(a) = 4

$3a - 5 = 4 \Rightarrow 3a = 4 + 5$

$\Rightarrow 3a = 9$

$\Rightarrow a = \frac{9}{3}$

$\Rightarrow a = 3$

(1, b) means the image of 1 is b.   That is f(1) = b

B = -2 $\Rightarrow 3(1) - 5$

$\Rightarrow 3 - 5$

= -2

***18.** The distance S (in kms) travelled by a particle in time 't' hours is given by S(t) = $\frac{t^2+t}{2}$ Find the distance travlled by the particle after.*

(i) three and half hours.   (ii) eight hours and fifteen minutes.

Answer:

*The distance travelled by the particle in time t hours is given by **S(t) =** $\frac{t^2+t}{2}$*

(i) t = 3.5 hours. Therefore, S(3.5) = $\frac{3.5^2 + 3.5}{2} = \frac{12.25+3.5}{2} = \frac{15.755}{2}$ *= 7.875*

The distance travlled in 3.5 hours is 7.875 Kms.

(ii) t = 8.25 hours. Therefore, S(8.25) = $\frac{8.25^2 + 8.25}{2} = \frac{68.0625+8.25}{2} = \frac{76.3125}{2}$ *= 38.15625*

The distance travelled in 8.25 hours is 38.16 Kms, approximately.

19.** If the function f : R → R defined by f(x) =* $\begin{cases} 2x+7, & x < -2 \\ x^2 - 2, & -2 \le x < 3 \\ 3x-2, & x \ge 3 \end{cases}$ ***Find the values of (i) f(4) (ii) f(-2) (iii) f(4)+ 2f(1) (iv) $\frac{f(1) - 3f(4)}{f(-3)}$

Answer:

(i) f(4) = 3x – 2   = 3(4) – 2   = 12 – 2   = 10

(ii) f(-2) = x^2- 2   = $(-2)^2$ – 2   = 4 – 2   = 2

(iii) f(4) + 2 f(1)

f(4) = 3x – 2   = 3(4) – 2   = 12 – 2   = 10

2f(1) = x^2 – 1   = 2 [$(1)^2$ – 2]   = 2(1 – 2)   = 2 (-1)   = -2

f(4) + 2 f(1) = 10 – 2   = 8

(iv) $\frac{f(1) - 3f(4)}{f(-3)}$

f(1) = -1,   f(4) = 10,

f(-3) = 2x+ 7   = 2(-3) + 7   = -6 + 7   = 1

$\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = \frac{-1-30}{1} = \frac{-31}{1}$ *= -31*

20.** Let f : A → B be a function defined by f(x) =* $\frac{x}{2} - 1$*, wehre A = { 2, 4, 6, 10, 12 }, B = { 0, 1, 2, 4, 5, 9 }. Respresent f by

(i) Set of ordered pairs,   (ii) a table

(iii) a graph   (iv) an arrow diagram

Answer:

Given :

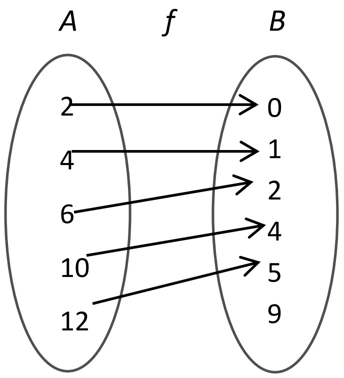
f(x) = $\frac{x}{2} - 1$*, A = { 2, 4, 6, 10, 12 } and B = { 0, 1, 2, 4, 5, 9 }*

$f(2) = \frac{2}{2} - 1 \quad = 1 - 1 \quad = 0$ $\qquad$ $f(4) = \frac{4}{2} - 1 \quad = 2 - 1 \quad = 1$

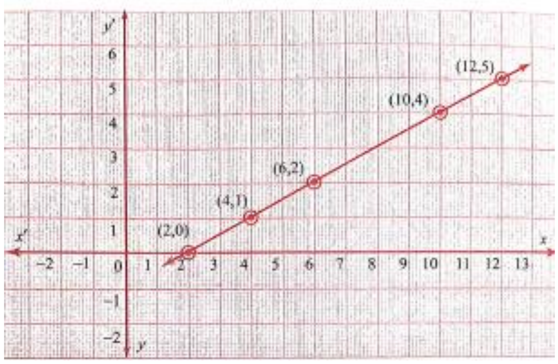
$f(6) = \frac{6}{2} - 1 \quad = 3 - 1 \quad = 2$ $\qquad$ $f(10) = \frac{10}{2} - 1 = 5 - 1 \quad = 4$

$f(12) = \frac{12}{2} - 1 = 6 - 1 \quad = 5$

(i) An arrow diagram



(iv) graphical form



(ii) a table

x	2	4	6	10	12
f(x)	0	1	2	4	5

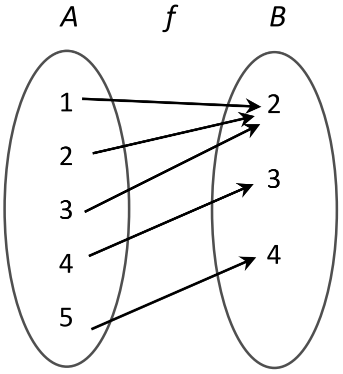
(iii) Set of ordered pairs $\qquad$ *f = { (2,0), (4,1), (6,2), (10,4), (12, 5)}*

21. Represent the function f = { (1, 2), (2, 2), (3, 2), (4, 3), (5, 4) } through
(i) an arrow diagram (ii) a table form (iii) a graph

Answer:

Given : f = { (1, 2), (2, 2), (3, 2), (4, 3), (5, 4) }

(i) An arrow diagram



(iii) graphical form

(ii) a table

x	1	2	3	4	5
f(x)	2	2	2	3	4

22. Show that the function $f : N \rightarrow N$ defined by $f(x) = 2x - 1$ is one – one but not onto.

Answer:

Given : $f(x) = 2x - 1$

$N = \{ 1, 2, 3, 4, 5, \ldots\ldots\ldots\}$

$f(1) = 2(1) - 1 \quad = 2 - 1 \quad = 1$

$f(2(= 2(2) - 1 \quad = 4 - 1 \quad = 3$

$f(3) = 2(3) - 1 \quad = 6 - 1 \quad = 5$

$f(4) = 2(4) - 1 \quad = 8 - 1 \quad = 7$

$f(5) = 2(5) - 1 \quad = 10 - 1 \quad = 9$

In the figure, for different elements in x, there are different image in f(x)
Hence $f : N \rightarrow N$ is said to be onto fuction if the range of f is equal to the co – domain of f.
But here the range is not equal to co – domain. Therefore it is one – one but not onto function.

23. Show that the function $f : N \rightarrow N$ defined by $f(m) = m^2 + m + 3$ is one – one function.

Answer:

Given : function $f : N \rightarrow N$ defined by $f(m) = m^2 + m + 3$

$N = \{ 1, 2, 3, 4, 5, 6, \ldots\ldots\ldots\ldots\ldots\ldots\},\ m \in N$

$f(m) = m^2 + m + 3$

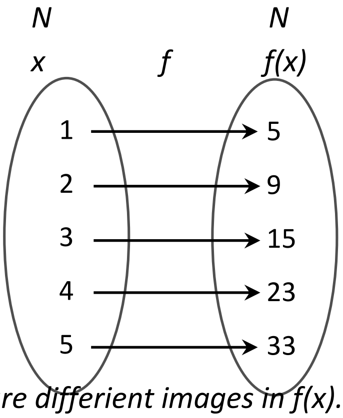
$f(1) = 1^2 + 1 + 3 \quad = 1 + 1 + 3 \quad = 5$

$f(2) = 2^2 + 2 + 3 \quad = 4 + 2 + 3 \quad = 9$

$f(3) = 3^2 + 3 + 3 \quad = 9 + 3 + 3 \quad = 15$

$f(4) = 4^2 + 4 + 3 \quad = 16 + 4 + 3 \quad = 23$

$f(5) = 5^2 + 5 + 3 \quad = 25 + 5 + 3 \quad = 33$



In the figure, for different elements in the (x) domain, there are differient images in f(x). Hence $f : N \rightarrow N$ is a one – one but not onto function as the range of f is not equal to co – domain.

24. **Let $A = \{ 1, 2, 3, 4 \}$ and $B = N$. Let $f : A \rightarrow B$ be defined by $f(x) = x^3$ then,**
(i) find the range of f (ii) identify the type of function.

Answer:

Given : $A = \{ 1, 2, 3, 4 \}$, $B = N$ and $f : A \rightarrow B$ be defined by $f(x) = x^3$

$f(x) = x^3$

$f(1) = 1^3 \quad = 1$ (i) Range of $f = \{ 1, 8, 27, 64\}$

$f(2) = 2^3 \quad = 8$ (ii) It is one – one and into function.

$f(3) = 3^3 \quad = 27$

$f(4) = 4^3 \quad = 64$

25. In each of the following cases state whether the function is bijective or not. Justify your answer. (i) $f : R \rightarrow R$ defined by $f(x) = 2x + 1$ (ii) $f : R \rightarrow R$ defined by $f(x) = 3 - 4x^2$

Answer:

(i) Given : $f : R \rightarrow R$ defined by $f(x) = 2x + 1$

$f(x) = 2x + 1$

$f(1) = 2(1) + 1 \quad = 2 + 1 \quad = 3 \qquad f(2) = 2(2) + 1 \quad = 4 + 1 \quad = 5$

$f(-1) = 2(-1) + 1 = -2 + 1 = -1 \qquad f(-2) = 2(-2) + 1 \quad = -4 + 1 \quad = -3$

It is a bijective function. Distinct element of A have distinct images in B and every element in B has a pre – image in A.

(ii) Given : f : R → R defined by $f(x) = 3 – 4x^2$

$f(x) = 3 – 4x^2$

$f(1) = 3 – 4(1)^2 = 3 – 4(1) = 3 – 4 = -1$ $f(2) = 3 – 4(2)^2 = 3 – 4(4) = 3 – 16 = -13$

$f(-1) = 3 – 4(-1)^2 = 3 – 4(1) = 3 – 4 = -1$ $f(-2) = 3 – 4(-2)^2 = 3 – 4(4) = 3 – 16 = -13$

It is not bijective function as the +ve numbers in R do not have pre image in X in R.

26. Let A = { -1, 1 } and B = { 0, 2 }. If the function f : A → B defined by f(x) = ax + b is an onto function? Find 'a' and 'b'

Answer :

Given : A = { -1, 1 } and B = { 0, 2 }

f(x) = ax + b is onto function.

f(-1) = 0 ⇒ a(-1) + b = 0 ⇒ -a + b = 0 -------------- (1)

f(1) = 2 ⇒ a(1) + b = 2 ⇒ a + b = 2 ----------------(2)

Solve (1) and (2)

$$\begin{array}{r} -a + b = 0 \\ \underline{a + b = 2} \\ 2b = 2 \\ b = 1 \end{array}$$

Substitute 'b' value in (1) or (2)

-a + b = 0 ⇒ -a + 1 = 0 ⇒ -a = -1

a = 1

Therefore a = 1 and b = 1

*27. **If the function f is defined by f(x) =*** $\begin{cases} x+2, & x>1 \\ 2, & -1\le x\le 1 \\ x-1, & -3<x<-1 \end{cases}$ ***Find the values of***

(i) f(3) (ii) f(0) (iii) f(-1.5) (iv) f(2) + f (-2)

Answer:

(i) f(3) = x + 2 = 3 + 2 = 5 (ii) f(0) = 2

(iii) f(-1.5) = x – 1 = (-1.5) – 1 = -2.5

(iv) f(2) + f(-2)

f(2) = x + 2 = 2 + 2 = 4

f(-2) = x – 1 = (-2) – 1 = -2 -1 = -3

f(2) + f(-2) = 4 – 3 = 1

28. A function f : [-5, 9] → R is defind as follows f(x) = $\begin{cases} 6x+1 & if\ -5\le x<2 \\ 5x^2-1 & if\ 2\le x<6 \\ 3x-4 & if\ 6\le x\le 9 \end{cases}$ ***Find the values of***

(i) f(-3) + f(2) (ii) f(7) – f(1) (iii) 2f(4) + f(8) (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Answer:

(i) f(-3) + f(2)

f(-3) = 6x + 1 = 6(-3) + 1 = -18 + 1 = -17

$f(2) = 5x^2 - 1$ $= 5(2)^2 – 1$ = 5(4) – 1 = 20 – 1 = 19

f(-3) + f(2) = -17 + 19 = 2

(ii) $f(7) - f(1)$

$f(7) = 3x - 4 \quad = 3(7) - 4 \quad = 21 - 4 \quad = 17$

$f(1) = 6x + 1 \quad = 6(1) + 1 \quad = 6 + 1 \quad = 7$

$f(7) - f(1) = 17 - 7 \quad = 10$

(iii) $2f(4) + f(8)$

$2f(4) = 2(5x^2 - 1) \quad = 2(5(4)^2 - 1) \quad = 2(5(16) - 1) \quad = 2(80 - 1) = 2(79) \quad = 158$

$f(8) = 3x - 4 \quad = 3(8) - 4 \quad = 24 - 4 \quad = 20$

$2f(4) + f(8) \quad = 158 + 20 \quad = 178$

(iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)}$

$f(-2) = 6x + 1 \quad = 6(-2) + 1 \quad = -12 + 1 \quad = -11$

$f(6) = 3x - 4 \quad = 3(6) - 4 \quad = 18 - 4 \quad = 14$

$f(4) = 5x^2 - 1 \quad = 5(4)^2 - 1 \quad = 5(16) - 1 \quad = 80 - 1 \quad = 79$

$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 - 11} = \frac{-22 - 14}{68} = \frac{-36}{68} = \frac{-9}{17}$$

29. The distance S an object travels under the influence of gravity in time 't' seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration dut to gravity), 'a', 'b' are constants. Check if the function S(t) is one – one.

$S(t) = \frac{1}{2}gt^2 + at + b$

Let t = 1, 2, 3, …………………seconds

$S(1) = \frac{1}{2}g(1)^2 + a(1) + b \quad = \frac{1}{2}g + a + b$

$S(2) = \frac{1}{2}g(2)^2 + a(2) + b \quad = 2g + 2a + b$

Yes, for every different values of t, there will be different values as images. And there will be diffeirent pre images for the different values of the range. Therefore it is one – one function.

30. The function 't' which mapes temperature in Celsius (C) into temperature in Fahrenheit (F) is defind by t(c) = F where F = $\frac{9}{5}C + 32$. Find,

(i) t(0) (ii) t(28) (iii) t(-10) (iv) The value of C when t(C) = 212

(v) the temperature when the Celsius value is equal to the Farenheit value.

Answer :

(i) Given : t(C) = F

F = t(C) $\quad t(c) = \frac{9}{5}C + 32$

$t(0) = \frac{9}{5}(0) + 32 \quad = 0 + 32 \quad = 32^0 F$

(ii) $t(28) = \frac{9}{5}(28) + 32 \quad = \frac{252}{5} + 32 \quad = 50.4 + 32 \quad = 82.4^0F$

(iii) $t(-10) = \frac{9}{5}(-10) + 32 \quad = \frac{-90}{5} + 32 \quad = -18 + 32 \quad = 14^0F$

(iv) $t(C) = 212$

$\frac{9}{5}C + 32 = 212 \quad \Rightarrow \frac{9}{5}C = 212 - 32 \quad \Rightarrow \frac{9}{5}C = 180$

$C = 180 \times \frac{5}{9} \quad \Rightarrow 20 \times 5 \quad = 100$

$C = 100^0c$

(v) $t(-40) = \frac{9}{5}(-40) + 32 \quad \Rightarrow 9(-8) + 32 \quad \Rightarrow -72 + 32$

$t(-40) = -40^0$

31. Find (f o g) and (g o f) when $f(x) = 2x + 1$ and $g(x) = x^2 – 2$

Answer :

Given : $f(x) = 2x + 1$, $g(x) = x^2 – 2$

(i) $fog = f(g(x))$ $= f(x^2 – 1)$ $= 2(x^2 – 2) + 1$ $= 2x^2 – 4 + 1$

$fog = 2x^2 – 3$

(ii) $gof = g(f(x))$ $= g(2x + 1)$ $= (2x + 1)^2 – 2$ $= (2x)^2 + 2(2x)(1) + 1^2 – 2$

$= 4x^2 + 4x + 1 – 2$ $= 4x^2 + 4x – 1$

gof $= 4x^2 + 4x – 1$

32. Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Answer :

Let $f_2(x) = 2x^2 – 5x + 3$ and $f_1(x) = \sqrt{x}$

$f(x) = \sqrt{2x^2 - 5x + 3}$ $= \sqrt{f_2(x)}$

$= f_1(f_2(x))$

$= f_1f_2(x)$

33. If $f(x) = 3x – 2$, $g(x) 2x + k$ and if (fog) = (gof), then find the value of 'k'.

Answer :

Given : $f(x) = 3x – 2$, $g(x) = 2x + k$ and (fog) = (gof)

$fog = f(g(x))$ $= f(2x + k)$ $= 3(2x + k) – 2$ $= 6x + 3k – 2$

$fog = 6x + 3k – 2$

$gof = g(f(x))$ $= g(3x – 2)$ $= 2(3x – 2) + k$ $= 6x – 4 + k$

$gof = 6x – 4 + k$

$fog = gof$ $\Rightarrow 6x + 3k – 2 = 6x -4 + k$

$\Rightarrow 3k – 2 = -4 + k$

$\Rightarrow 3k – k = -4 + 2$

$\Rightarrow 2k = -2$

Therefore k = -1

34. Find 'k' if $fof(k) = 5$ where $f(k) = 2k – 1$.

Answer :

Given : $fof(k) = 5$ and $f(k) = 2k – 1$.

$fof(k) = f(f(k))$ $= 2(2k – 1) – 1$

$= 4k – 2 – 1$

$= 4k – 3$

Thus $fof(k) = 4k – 3$

But, it is given that $fof(k) = 5$

Therefore $4k – 3 = 5$ $\Rightarrow 4k = 5 + 3$

$4k = 8$ $\Rightarrow k = \frac{8}{4}$ $\Rightarrow$ **K = 2**

35. Find 'x' if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$

Answer :

Given : $gff(x) = fgg(x)$, $f(x) = 3x + 1$ and $g(x) = x + 3$

$gff(x) = g[f\{f(x)\}]$ $\Rightarrow g[f(3x + 1)]$ $\Rightarrow g[3(3x+1)+1]$ $\Rightarrow g(9x+4)$

$\Rightarrow (9x+4) + 3 \quad \Rightarrow 9x + 7$

$Fgg(x) = f[g\{g(x)\}] \quad \Rightarrow f[g(x + 3)] \quad \Rightarrow f[(x + 3)+3] \quad \Rightarrow f(x + 6)$

$\Rightarrow [3(x + 6) + 1] \Rightarrow (3x + 18 + 1) \quad \Rightarrow 3x + 19$

These two quantities being equal, we get

$9x + 7 = 3x + 19 \quad \Rightarrow 9x - 3x = 19 - 7 \quad \Rightarrow 6x - 12$

That is $x = 2$

36. Find (f o g) and (g o f) when f(x) = x – 6 and g(x) = x^2

Answer :

Given : $f(x) = x - 6$ and $g(x) = x^2$

$fog = f(g(x)) \quad \Rightarrow f(x^2) \quad \Rightarrow x^2 - 6$

$fog = x^2 - 6$

$gof = g(f(x)) \quad \Rightarrow g(x - 6) \quad \Rightarrow (x - 6)^2$

$gof = x^2 - 12x + 36$ *Therefore* $fog \neq gof$

37. Find (f o g) and (g o f) when f(x) = 3 + x and g(x) = x – 4

Answer :

Given : $f(x) = 3 + x$ and $g(x) = x - 4$

$fog = f(g(x)) \quad \Rightarrow f(x - 4) \quad \Rightarrow 3 + x - 4$

$fog = x - 1$

$gof = g(f(x)) \quad \Rightarrow g(3 + x) \quad \Rightarrow (3 + x) - 4$

$gof = x - 1$ *Therefore* $fog = gof$

38. Find (f o g) and (g o f) when f(x) = $\frac{2}{x}$ and g(x) = $2x^2 - 1$

Answer :

Given : $f(x) = \frac{2}{x}$ and $g(x) = 2x^2 - 1$

$fog = f(g(x)) \quad \Rightarrow f(2x^2 - 1) \quad \Rightarrow \frac{2}{2x^2 - 1}$

$fog = \frac{2}{2x^2 - 1}$

$gof = g(f(x)) \quad \Rightarrow g\left(\frac{2}{x}\right) \quad \Rightarrow 2\left(\frac{2}{x}\right)^2 - 1 \Rightarrow 2\left(\frac{4}{x^2}\right) - 1 \quad \Rightarrow \left(\frac{8}{x^2}\right) - 1$

$gof = \frac{8}{x^2} - 1$ *Therefore* $fog \neq gof$

39. Find (f o g) and (g o f) when f(x) = $\frac{x+6}{3}$ and g(x) = 3 – x

Answer :

Given : $f(x) = \frac{x+6}{3}$ and $g(x) = 3 - x$

$fog = f(g(x)) \quad \Rightarrow f(3 - x) \quad \Rightarrow \frac{3-x+6}{3} \quad \Rightarrow \frac{9-x}{3}$

$fog = \frac{9-x}{3}$

$gof = g(f(x)) \quad \Rightarrow g\left(\frac{x+6}{3}\right) \quad \Rightarrow 3 - \left(\frac{x+6}{3}\right) \Rightarrow \frac{9-x-6}{3}$

$gof = \frac{3-x}{3}$ *Therefore* $fog \neq gof$

40. Find the value of k, such that fog = gof f(x) = 3x + 2 and g(x) = 6x – k

Answer :

Given : $f(x) = 3x + 2$ and $g(x) = 6x - k$

$fog = f(g(x)) \quad \Rightarrow f(6x - k) \quad \Rightarrow 3(6x - k) + 2 \quad \Rightarrow 18x - 3k + 2$

$fog = 18x - 3k + 2$

$gof - g(f(x)) \quad \Rightarrow g(3x + 2) \quad \Rightarrow 6(3x + 2) - k \quad \Rightarrow 18x + 12 - k$

$gof = 18x + 12 - k$

$fog = gof \quad \Rightarrow 18x - 3k + 2 = 18x + 12 - k$

$\Rightarrow -3k + 2 = 12 - k$

$\Rightarrow -3k + k = 12 - 2$

$\Rightarrow -2k = 10$

$\Rightarrow k = -5$

Therefore k = -5

41. ***Find the value of k, such that fog = gof, $f(x) = 2x - k$ and $g(x) = 4x + 5$***

Answer:

Given : $f(x) = 2x - k$ *and* $g(x) = 4x + 5$

$fog = f(g(x)) \quad \Rightarrow f(4x + 5) \quad \Rightarrow 2(4x + 5) - k$

$fog = 8x + 10 - k$

$gof = g(f(x)) \quad \Rightarrow g(2x - k) \quad \Rightarrow 4(2x - k) + 5$

$gof = 8x - 4k + 5$

$fog = gof \quad \Rightarrow 8x + 10 - k = 8x - 4k + 5$

$\Rightarrow 10 - k = -4k + 5$

$\Rightarrow -k + 4k = 5 - 10$

$\Rightarrow 3k = -5$

$\Rightarrow k = \frac{-5}{3}$

42. ***If f(x) 2x – 1 and g(x) = $\frac{x+1}{2}$ show that fog = gof = x***

Answer :

Given : $f(x) = 2x - 1$ *and* $g(x) = \frac{x+1}{2}$

$fog = f(g(x)) \quad \Rightarrow f\left(\frac{x+1}{2}\right) \quad \Rightarrow 2\left(\frac{x+1}{2}\right) - 1$

$fog = x + 1 - 1 = x$

$fog = x$ --------------------(1)

$gof = g(f(x)) \quad \Rightarrow g(2x - 1) \quad \Rightarrow \frac{2X-1+1}{2}$

$gof = \frac{2X}{2}$ *Therefore fog = gof*

$gof = x$ --------------------(2)

From (1) and (2) LHS = RHS

Therefore fog = gof

43. ***If $f(x) = x^2 - 1$, $g(x) = x - 2$ find 'a', if gof(a) = 1.***

Answer:

Given : $f(a) = a^2 - 1$ *and* $g(a) = a - 2$

$gof = g(f(x)) \quad \Rightarrow g(a^2 - 1) \quad \Rightarrow (a^2 - 1) - 2$

$\Rightarrow a^2 - 1 - 2 \quad \Rightarrow a^2 - 3$

$Gof(a) = 1 \quad \Rightarrow a^2 - 3 = 1 \quad \Rightarrow a^2 = 1 + 3$

$\Rightarrow a^2 + 4$ $\Rightarrow a = \sqrt{4}$

$\Rightarrow a = \pm 2$

44. ***Find 'k', if f(k) = 2k – 1 and fof(k) = 5***

Answer:

Given : $f(k) = 2k - 1$

$fof = f(f(k))$ $\Rightarrow f(2k - 1)$ $\Rightarrow 2(2k - 1) - 1$

$fof = 4k - 2 - 1$

$fof = 4k - 3$

$fof(k) = 5$ $\Rightarrow 4k - 3 = 5$ $\Rightarrow 4k = 5 + 3$

$\Rightarrow 4k = 8$ $\Rightarrow k = \frac{8}{4}$

$\Rightarrow k = 2$

45. ***Let A, B, C $\subseteq$ N and a function f : A $\to$ B be defined by f(x) 2x + 1 and g:B $\to$ C be defined by g(x) = x^2. Find the range of fog and gof***

Answer:

Given : $f(x) = 2x + 1$ and $g(x)\ x^2$

$fog = fog(x)$ $\Rightarrow f(x^2)$ $\Rightarrow 2(x^2) + 2$

$fog = 2x^2 + 1$

$gof = g(f(x))$ $\Rightarrow g(2x + 1)$ $\Rightarrow (2x + 1)^2$

$\Rightarrow 4x^2 + 4x + 1$

$gof = 4x^2 + 4x + 1$

Range of fog = $\{\, y \mid y = 2x^2 + 1, x \in N\}$

Range of gof = $\{\, y \mid y = 4x^2 + 4x + 1, x \in N \}$

46. ***Let f(x) = x^2 – 1. Find fof and fofof***

Answer :

Given : $f(x) = x^2 - 1$

(i) $fof = f(f(x))$ $\Rightarrow f(x^2 - 1)$ $\Rightarrow (x^2 - 1)^2 - 1$ $\Rightarrow x^4 - 2x^2 + 1 - 1$

$\Rightarrow x^4 - 2x^2$

(ii) $fofof = fof(f(x))$ $\Rightarrow f(x^4 - 2x^2)$ $\Rightarrow (x^4 - 2x^2)^2 - 1$ $\Rightarrow x^8 - 4x^2 + 4x^4 - 1$

47. ***If f : R $\to$ R and g : R $\to$ R are defined by f(x) = x^5 and g(x) = x^4 then check if f, g are one – one and fog is one – one?***

Answer :

Given : $f(x) = x^5$ and $g(x) = x^4$

$fog = f(g(x))$ $\Rightarrow f(x^4)$ $\Rightarrow (x^4)^5$ $\Rightarrow x^{20}$

Therefore f is one – one also gof is one – one.

48. ***If f(x) = x – 1, g(x) = 3x + 1 and h(x) = x^2 then prove that fo(goh) = (fog)oh***

Answer :

Given : $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

LHS : fo(goh)

$goh = g(h(x))$ $\Rightarrow g(x^2)$ $\Rightarrow 3x^2 + 1$

$goh = 3x^2 + 1$

$fo(goh) = f(3x^2 + 1)$ $\Rightarrow 3x^2 + 1 - 1$ $\Rightarrow 3x^2$

fo(goh) = $3x^2$ ------------------ (1)

RHS: (fog)oh

fog = f(g(x)) ⇒f(3x + 1) ⇒ 3x + 1 – 1 ⇒3x

fog = 3x

(fog)oh= (fog)(h(x) ⇒ (fog)(x^2) ⇒ $3x^2$

(fog)oh = $3x^2$ ------------------ (2)

From (1) and (2)

LHS = RHS That is

fo(goh) = (fog)oh

49. If f(x) = x^2, g(x) = 2x and h(x) = x + 4 then prove that fo(goh) = (fog)oh

Answer :

Given : f(x) x^2, g(x) = 2x and h(x) = x + 4

goh = g(h(x)) ⇒ g(x +4) ⇒ 2 (x + 4) ⇒ 2x + 8

goh = 2x + 8

fo(goh) = f(2x + 8) ⇒$(2x + 8)^2$ ⇒ $4x^2 + 32x + 64$

fo(goh) = $4x^2 + 32x + 64$ ------------------ (1)

RHS: (fog)oh

fog = f(g(x)) ⇒f(2x) ⇒ $(2x)^2$ ⇒ $4x^2$

fog = $4x^2$

(fog)oh= (fog)(h(x) ⇒ (fog)(x + 4) ⇒ $4\ (x+4)^2$ ⇒ $4(x^2 + 8x + 16\ )$ ⇒ $4x^2 + 32x + 64$

(fog)oh = $4x^2 + 32x + 64$------------------ (2)

From (1) and (2)

LHS = RHS That is

fo(goh) = (fog)oh

50. If f(x) = x – 4, g(x) = x^2 and h(x) = 3x – 5 then prove that fo(goh) = (fog)oh

Answer :

Given : f(x) x – 4 , g(x) = x^2 and h(x) = 3x – 5

goh = g(h(x)) ⇒ g(3x – 5) ⇒ $(3x – 5)^2$ ⇒ $9x^2 – 30x + 25$

gog = $9x^2 – 30x + 25$

fo(goh) = f($9x^2 – 30x + 25$) ⇒ ($9x^2 – 30x + 25$) – 4 ⇒ $9x^2 – 30x + 25 – 4$

fo(goh) = $9x^2 – 30x + 21$ ------------------ (1)

RHS: (fog)oh

fog = f(g(x)) ⇒f(x^2) ⇒ $x^2 – 4$

fog = $x^2 – 4$

(fog)oh= (fog)(h(x) ⇒ (fog)(3x – 5) ⇒ $(3x – 5)^2 – 5$ ⇒ $9x^2 – 30x + 25 – 4$

(fog)oh = $9x^2 – 30x + 21$------------------ (2)

From (1) and (2)

LHS = RHS That is

fo(goh) = (fog)oh

51. If f(x) = 2x + 3, g(x) = 1 – 2x and h(x) = 3x. Prove that fo(goh) = (fog)oh

Answer :

Given : f(x) = 2x + 3, g(x) = 1 – 2x and h(x) = 3x

LHS : fo(goh)

goh $= g(h(x))$ $\Rightarrow g(3x)$ $\Rightarrow 1 - 2(3x)$ $\Rightarrow 1 - 6x$

goh $= 1 - 6x$

fo(goh) $= f(1 - 6x)$ $\Rightarrow 2(1 - 6x) + 3$ $\Rightarrow 2 - 12x + 3$

fo(goh) $= 5 - 12x$ ------------------ (1)

RHS: (fog)oh

fog $= f(g(x))$ $\Rightarrow f(1 - 2x)$ $\Rightarrow 2(1 - 2x) + 3$ $\Rightarrow 2 - 4x + 3$

fog $= 5 - 4x$

(fog)oh= (fog)(h(x) $\Rightarrow$ (fog)(3x) $\Rightarrow 5 - 4(3x)$

(fog)oh $= 5 - 12x$ ------------------ (2)

From (1) and (2)

LHS = RHS That is

fo(goh) = (fog)oh

52. Let f = { (-1, 3), (0, -1), (2, -9) } be a linear function from Z into Z. Find f(x).

Answer :

Given : **f = { (-1, 3), (0, -1), (2, -9) }**

$f(x) = ax + b$ is onto function.

$f(-1) = -3$ $\Rightarrow a(-1) + b = 3$ $\Rightarrow -a + b = 3$ -------------- (1)

$f(0) = -1$ $\Rightarrow a(0) + b = -1$ $\Rightarrow 0 + b = -1$ $\Rightarrow b = -1$

Substitute 'b' value in (1)

$-a + b = 0$ $\Rightarrow -a + (-1) = 3$ $\Rightarrow -a - 1 = 3$ $\Rightarrow -a = 3 + 1$ $\Rightarrow -a = 4$

$a = -4$

Therefore the linear function f(x) is $-4x - 1$

53. In electrical circuit theory, a circuit C(t) is called linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit C(t) = 3t is linear.

Answer :

Given : $C(t) = 3t$

To prove : C(t) is linear

$C(at_1) = 3at_1$ --------------- (1)

$c(bt_2) = 3bt_2$ ----------------(2)

(1) + (2)

$C(at_1) + c(bt_2) = 3at_1 + 3bt_2$ $\Rightarrow C(at_1 + bt_2) = 3(at_1 + bt_2)$

Superposition principle is satisified.

Hence $C(t) = 3t$ is linear function.

THEOREMS

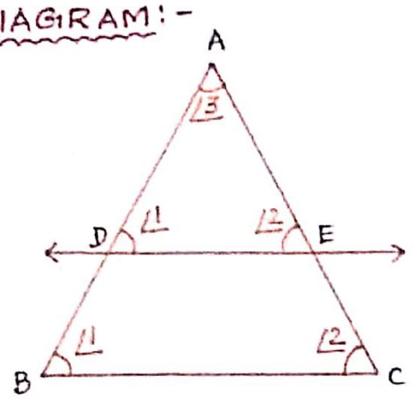
THEOREM – 1

BASIC PROPORTIONALITY THEOREM (OR) THALES THEOREM.

State and prove Basic proportionality theorme (or) Thales theorem.

Statement : *If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then divides the sides in the same ratio.*

DIAGRAM:-



Given : *In a ΔABC, D is a point on AB and E is a point on AC*

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: *Draw a line DE || BC*

Step. No.	*Statement*	*Reason*
1.	$\angle ABC = \angle ADE = \angle 1$------------(1)	*Corresponding angles*
2.	$\angle ACB = \angle AED = \angle 2$------------(2)	*Corresponding angles*
3.	$\angle BAC = \angle DAE = \angle 3$------------(3)	*Common angles to ΔABC and ΔADE*
4.	$\Delta ABC \sim \Delta ADE$	*AA Similarity, from (1), (2) and (3)*
5.	$\frac{AB}{AD} = \frac{AC}{AE}$	*corresponding sides are proportional.*
6.	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	*Split AB and AC*
7.	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	
8.	$\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$ *(Hence proved)*	*By cancelling 1 both sides.* *Taking their reciprocals*

CONVERSE OF THALES THEOREM (or) BASIC PROPORTIONALITY THEOREM.

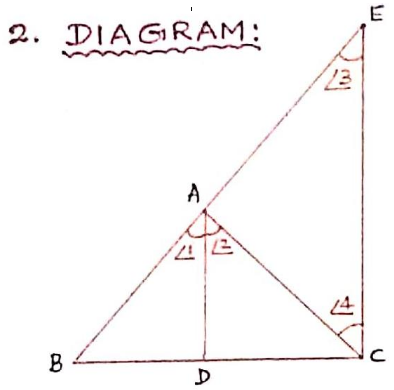
If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

THEOREM – 2

ANGLE BISECTOR THEOREM.

State and prove Angle Bisector Theorem.

Statement : *The internal bisector of an angle of triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.*



Given : *In a Δ ABC, AD is the internal bisector of $\angle$BAC which meets BC at D*

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : *Draw CE || DA to meet BA produced at E.*

Let $\angle$BAD = $\angle$1, $\angle$DAC = $\angle$2, $\angle$AEC = $\angle$3, $\angle$ACE = $\angle$4

Proof:

Step. No.	**Statement**	**Reason**
1.	*$\angle$1 = $\angle$2 --------------------(1)*	*AD is the angle bisector of $\angle$A*
2.	*$\angle$1 = $\angle$3 --------------------(2)*	*Corresponding angles. Since CE || DA*
3.	*$\angle$2 = $\angle$4 --------------------(3)*	
4.	*$\angle$3 = $\angle$4*	*Alternate angles, because AC is transversal* *From (1), (2) and (3)*
5.	*AE = AC ---------------------(4)*	*Sides opposite to equal angles are equal* *By thales theorem.*
6.	$\therefore \frac{BD}{DC} = \frac{AB}{AE}$	
7.	$\therefore \frac{BD}{DC} = \frac{AB}{AC}$ *(Hence proved)*	*From (4)*

CONVERSE OF ANGLE BISECTOR THEOREM.

If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.

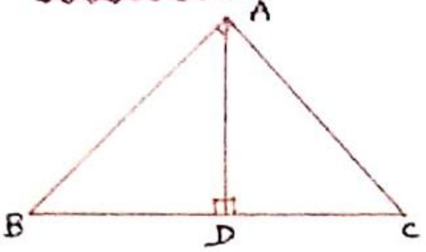
THEOREM – 3

PYTHAGORAS THEOREM

State and prove Pythagoras Theorem.

Statement : *In a rightagnle triange square hypotenuse is equal to the sum of square of other two sides.*

2. DIAGRAM:



Given : *In a ΔABC, $\angle A = 90^0$*
To Prove: *$AB^2 + AC^2 = BC^2$*
Construction: *Draw $AD \perp BC$*

Proof:

Step. No.	***Statement***	***Reason***
1.	$\Delta ABC \sim \Delta DBA$	*AA similarity, because, $\angle B$ is common,* *$\angle BAC = \angle BDA = 90^0$*
2	$\frac{AB}{BD} = \frac{BC}{AB}$	*Corresponding sides are proportional.*
	$AB^2 = BC \text{ x } BD$ ------------(1)	
3.	$\Delta ABC \sim \Delta DAC$	*AA similarity, because, $\angle C$ is common,* *$\angle BAC = \angle ADC = 90^0$*
4.	$\frac{BC}{AC} = \frac{AC}{DC}$	*Corresponding sides are proportional.*
	$AC^2 = BC \text{ x } DC$ ------------(2)	
5.	*$AB^2 + AC^2$ =(BC x BD) + (BC x DC)* *= BC x (BD + DC)* *= BC x BC* *$AB^2 + AC^2 = BC^2$ (hence proved)*	*By adding (1) and (2)*

CONVERSE OF PYTHAGORAS THEOREM.

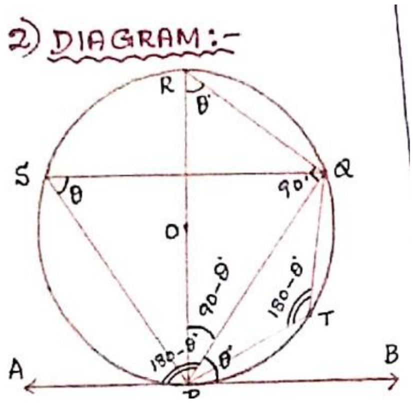
If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is right angle triangle.

THEOREM – 4

ALTERNATE SEGMENT THEOREM (OR) TANGENT – CHORD THEOREM.

State and prove Alternate segment theorem (or) Tangent – chord theorem..

Statement : *If from the point of contact of a tangent of a circle, a chord is drawn then the angles between the tangent and the chord are respectively to the angles in the corresponding alternate segments.*



Given : *A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.*

To Prove : $\angle QPB = \angle PSQ$, $\angle QPA = \angle PTQ$

Construction : *Draw the diameter POR.*
Draw QR, QS, QT, PS and PT

Proof:

Step. No.	Statement	Reason
1.	$\angle QPB = \theta$ (say) $\therefore \angle QPR = 90 - \theta$-----(1)	Diameter RP is perpendicular to tangent AB
2.	$\angle PQR = 90^0$	Angle in semicircle is 90^0
3.	$\angle QRP = \theta$--------------------(2)	In Δ PQR, the sum of all angles are 180^0 so $(90 - \theta) + 90 + \angle QRP = 180^0$
4.	$\angle QRP = \angle QSP = \theta$--------------(3)	Angles in the same segment are equal
5.	$\angle QPB = \angle QSP = \theta$ (hence (i) proved)	From (1), (2) and (3)
6.	$\angle QPB = \theta \Rightarrow \angle QPA = 180 - \theta$------- (4)	Linear pair of angles
7.	$\angle QSP = \theta \Rightarrow \angle PTQ = 180 - \theta$--------(5)	Sum of opposite angles of a cyclic quadrilateral PTQS is 180^0
8.	$\angle QPA = \angle PTQ$	From (4) and (5)

COORDINATE GEOMETRY

1. ***Find the area of the triangle whose vertices are (-3, 5), (5, 6) and (5, -2)***

Answer:

Area of the triangle $=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

$x_1 = -3$, $x_2 = 5$, $x_3 = 5$, $y_1 = 5$, $y_2 = 6$ and $y_3 = -2$

Area of the triangle $=\frac{1}{2}\begin{pmatrix} -3 & 5 & 5 & -3 \\ 5 & 6 & -2 & 5 \end{pmatrix}$ $=\frac{1}{2}[\,(-18-10+25)-(25+30+6)$

$=\frac{1}{2}[\,(-3)-(61)]$ $=\frac{1}{2}(-3-61)$ $=\frac{1}{2}(-64)=-32$

Area of the triangle *= 32 Sq. Units.*

2. ***Find the area of the triangle whose vertices are (1, -1), (-4, 6) and (-3. -5)***

Answer:

Area of the triangle $=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

$x_1 = 1$, $x_2 = -4$, $x_3 = -3$, $y_1 = -1$, $y_2 = 6$ and $y_3 = 5$

Area of the triangle $=\frac{1}{2}\begin{pmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{pmatrix}$ $=\frac{1}{2}[\,(6+20+3)-(4-18-5)]$

$=\frac{1}{2}[\,(29)-(-19)]$ $=\frac{1}{2}(29+19)$ $=\frac{1}{2}(48)=24$

Area of the triangle *= 24 Sq. Units.*

3. ***Find the area of the triangle whose vertices are (-10, -4), (-8, -1) and (-3, -5)***

Answer:

Area of the triangle $=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

$x_1 = -10$, $x_2 = -8$, $x_3 = -1$, $y_1 = -4$, $y_2 = -1$ and $y_3 = -5$

Area of the triangle $=\frac{1}{2}\begin{pmatrix} -10 & -8 & -3 & -10 \\ -4 & -1 & -5 & -4 \end{pmatrix}$ $=\frac{1}{2}[\,(10+40+12)-(32+3+50)]$

$=\frac{1}{2}[\,(62)-(85)]$ $=\frac{1}{2}(62-85)$ $=\frac{1}{2}(-23)=-11.5$

Area of the triangle *= 11.5 Sq. Units.*

4. ***Show that the points P (-1.5, 3), Q (6, -2), R (-3, 4) are collinear.***

Answer:

Area of the triangle $=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

$x_1 = -1.5$, $x_2 = 6$, $x_3 = -3$, $y_1 = 3$, $y = -2$ and $y_3 = -3$

Area of triangle PQR $=\frac{1}{2}\begin{Bmatrix} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{Bmatrix}$ $=\frac{1}{2}[\,(3+24-9)-(18+6-6)]$

$=\frac{1}{2}[\,(18)-(18)]$ $=\frac{1}{2}(18-18)$ $=\frac{1}{2}(\,0\,)=0$

Therefore the given points are collinear.

5. ***Determine whether the points are collinear. (a, b + c), (b, c + a), (c, a + b)***

Area of the triangle $=\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

$x_1 = a$, $x_2 = b$, $x_3 = c$, $y_1 = b+c$, $y_2 = c+a$ and $y_3 = a+b$

Area of triangle PQR $=\frac{1}{2}\begin{Bmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{Bmatrix}$

$= \frac{1}{2}[\ (a(c+a)+b(a+b)+c(b+c)) - (b(b+c)+c(c+a)+a(a+b)]$

$= \frac{1}{2}\ [\ (ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ac + a^2 + ab\ )]$

$= \frac{1}{2}\ (ac + a^2 + ab + b^2 + bc + c^2 - b^2 - bc - c^2 - ac - a^2 - ab\ ) \quad = \frac{1}{2}(\ 0\ ) = 0$

Therefore the given points are collinear.

6. ***Determine whether the points are collinear $\left(-\frac{1}{2}, 3\right)$, (-5, 6) and (-8, 8)***

Area of the triangle $= \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ *Sq. units.*

$\mathbf{x_1 = -\frac{1}{2},\quad x_2 = -5,\quad x_3 = -8,\quad y_1 = 3,\quad y_2 = 6}$ ***and*** $\mathbf{y_3 = 8}$

Area of triangle PQR $= \frac{1}{2}\left\{\begin{matrix} \frac{-1}{2} & -5 & -8 & \frac{-1}{2} \\ 3 & 6 & 8 & 3 \end{matrix}\right\} \qquad = \frac{1}{2}[\ (-3 - 40 - 24\ ) - (-15 - 48 - 4\ )]$

$= \frac{1}{2}\ [\ (-\ 67\ ) - (-67)] \qquad = \frac{1}{2}\ (-67 + 67\ ) \qquad = \frac{1}{2}(\ 0\ ) \qquad = 0$

Therefore the given points are collinear.

7. ***If the area of the triangle formed by the verticies A (-1, 2), B (k, -2) and C (7, 4) (taken in order) is 22 sq. units, find the value of 'k'.***

Answer:

$\mathbf{x_1 = -1,\quad x_2 = k,\quad x_3 = 7,\quad y_1 = 2,\quad y_2 = -2}$ ***and*** $\mathbf{y_3 = 4}$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\} = 22$ *Sq. Units*

$\Rightarrow \frac{1}{2}\begin{pmatrix} -1 & k & 7 & -1 \\ 2 & -2 & 4 & 2 \end{pmatrix} = 22$

$\Rightarrow \frac{1}{2}[\ (2 + 4k + 14\ ) - (2k - 14 - 4\ \ )] = 22$

$\Rightarrow \frac{1}{2}[\ (16 + 4k\ ) - (-18 + 2k)] = 22 \qquad \Rightarrow \frac{1}{2}[\ 16 + 4k + 18 - 2k] = 22$

$\Rightarrow 34 + 2k = 44 \qquad \Rightarrow 2k = 44 - 34 \qquad \Rightarrow 2k = 10$

$\Rightarrow\ k = \frac{10}{2} \qquad \mathbf{\Rightarrow k = 5}$

8. ***If the area of the triangle formed by the verticies A (0, 0), B (p, 8) and C (6, 2) (taken in order) is 20 sq. units, find the value of 'p'.***

Answer:

$\mathbf{x_1 = 0,\quad x_2 = p,\quad x_3 = 6,\quad y_1 = 0,\quad y_2 = 8}$ ***and*** $\mathbf{y_3 = 2}$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\} \qquad = 20$ *Sq. Units*

$\Rightarrow \frac{1}{2}\begin{pmatrix} 0 & p & 6 & 0 \\ 0 & 8 & 2 & 0 \end{pmatrix} \qquad = 20$

$\Rightarrow \frac{1}{2}[\ (0 + 2p + 0\ ) - (0 + 48 + 0\ \ )] = 20$

$\Rightarrow \frac{1}{2}[\ (2p\ ) - (48)] = 20 \qquad \Rightarrow \frac{1}{2}[\ 2p - 48]\ \ = 20$

$\Rightarrow 2p - 48 = 40 \qquad \Rightarrow 2P = 40 + 48 \qquad \Rightarrow 2P = 88$

$\Rightarrow\ P = \frac{88}{2} \qquad \mathbf{\Rightarrow P = 44}$

9. ***If the area of the triangle formed by the verticies A (p, p), B (5, 6) and C (5, -2) (taken in order) is 32 sq. units, find the value of 'p'.***

Answer:

$\mathbf{x_1 = p,\quad x_2 = 5,\quad x_3 = 5,\quad y_1 = p,\quad y_2 = 6}$ ***and*** $\mathbf{y_3 = 2}$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ = 32 Sq. Units

$\Rightarrow \frac{1}{2}\begin{pmatrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{pmatrix}$ = 32

$\Rightarrow \frac{1}{2}[\,(6p - 10 + 5p\,) - (5p + 30 - 2p\,)] = 32$

$\Rightarrow \frac{1}{2}[\,(11p - 10\,) - (3p + 30\,] = 32$ $\quad \Rightarrow \frac{1}{2}[\,11p - 10 - 3p - 30\ = 32$

$\Rightarrow 8p - 40\ = 64$ $\quad \Rightarrow 8p = 64 - 40$ $\quad \Rightarrow 8p = 104$

$\Rightarrow\ p = \frac{104}{2}$ $\quad$ $\Rightarrow$ **p = 13**

10. ***In each of the following, find the value of 'a' for which the given points are collinear***

(i) (2, 3), (4, a) and (6, -3) ***(ii) (a, 2 – 2a), (-a + 1, 2a) and (-4 – a, 6 – 2a)***

(i) (2, 3), (4, a) and (6, -3)

Answer:

$x_1 = 2,\quad x_2 = 4,\quad x_3 = 6,\quad y_1 = 3,\quad y_2 = a$ **and** $y_3 = -3$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ = 0 Sq. Units

$\Rightarrow \frac{1}{2}\begin{pmatrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{pmatrix}$ = 0

$\Rightarrow \frac{1}{2}[\,(2a - 12 + 18\,) - (12 + 6a - 6\,)] = 0$

$\Rightarrow \frac{1}{2}[\,(2a + 6\,) - (6 + 6a\,] = 0$ $\quad \Rightarrow \frac{1}{2}[\,2a + 6 - 6 - 6a] = 0$

$\Rightarrow\ -4a = 0$

$\Rightarrow\ a = \frac{0}{-4}$ $\quad$ $\Rightarrow$ **a = 0**

(ii) (a, 2 – 2a), (-a + 1, 2a) and (-4 – a, 6 – 2a)

Answer:

$x_1 = a,\quad x_2 = -a + 1,\quad x_3 = -4 - a,\quad y_1 = 2 - 2a,\quad y_2 = 2a$ **and** $y_3 = 6 - 2a$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ = 0 Sq. Units

$\Rightarrow \frac{1}{2}\begin{pmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{pmatrix}$ = 0

$\Rightarrow \frac{1}{2}[\,(2a^2 + (-a+1)(6-2a) + (-4-a)(2-2a)) - ((2-2a)(-a+1)+2a(-4-a)+(6-2a)a)] = 0$

$\Rightarrow \frac{1}{2}[\,(2a^2-6a+2a^2+6-2a-8+8a-2a+2a^2) - (-2a+2+2a^2-2a-8a-2a^2+6a-2a^2)\,] = 0$

$\Rightarrow \frac{1}{2}[\,2a^2 -6a+2a^2+6-2a-8+8a-2a+2a^2+2a-2-2a^2+2a+8a+2a^2-6a+2a^2\,] = 0$

$\Rightarrow \frac{1}{2}[\,8a^2 + 4a - 4\,] = 0$

$\Rightarrow 8a^2 + 4a - 4\ = 0$ $\quad \Rightarrow 2a^2 + a - 1 = 0$

$\Rightarrow\ (2a - 1\,)\,(\,a + 1\,) = 0$

$\Rightarrow 2a - 1 = 0$ $\quad \Rightarrow 2a = 1$ $\quad \Rightarrow$ $\mathbf{a = \frac{1}{2}}$

$\Rightarrow\ a + 1 = 0$ $\quad \Rightarrow$ **a = -1**

11. ***If the points P (-1, -4), Q (b, c) and R (5, -1) are collinear and if 2b + c = 4 then find the values of 'b' and 'c'.***

Answer:

$x_1 = -1,\quad x_2 = b,\quad x_3 = 5,\quad y_1 = -4,\quad y_2 = c$ **and** $y_3 = -1$

Area of the triangle ⇒ $\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *= 0 Sq. Units*

⇒ $\frac{1}{2}\begin{pmatrix} -1 & b & 5 & -1 \\ -4 & c & -1 & -4 \end{pmatrix}$ *= 0*

⇒ $\frac{1}{2}[(-c-b-20) - (-4b+5c+4)] = 0$

⇒ $\frac{1}{2}[(-c-b-20+4b-5c-4] = 0$ ⇒ $\frac{1}{2}[-6c-3b-24] = 0$

⇒ $-6c-3b-24 = 0$ ⇒ $-6c-3b = 24$

⇒ $-b-2c = 7$ -----------------(1)

⇒ $2b+c = 4$ --------------(2) (given)

By solving (1) and (2) we get b = 3 and c = -2

12. ***The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has verticies at (-3, 2), (-1, -1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.***

Answer:

Vertices of one triangular tile are at (-3, 2) (-1, -1) and (1, 2)

$x_1 = -3$, $x_2 = -1$, $x_3 = 1$, $y_1 = 2$, $y_2 = -1$ **and** $y_3 = 2$

Area of one tile $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ *Sq. units.*

Area of one tile $= \frac{1}{2}\begin{Bmatrix} -3 & -1 & 1 & -3 \\ 2 & -1 & 2 & 2 \end{Bmatrix}$ $= \frac{1}{2}[(3-2+2) - (-2-1-6)]$

$= \frac{1}{2}[(3) - (-9)]$ $= \frac{1}{2}(3+9)$ $= \frac{1}{2}(12)$ $= 6$

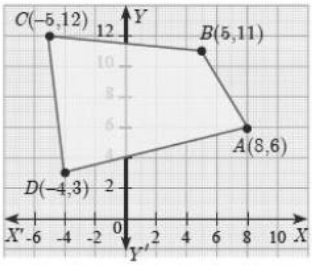
Therefore area of one tile = 6 sq. units.

Since the floor is covered by 110 triangle shaped identical tiles,

Area of floor = 110 x 6 = 660 sq. units.

13. ***Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (-5, 12) and (-4, 3)***

Answer:



Area of the triangle ⇒ $\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

$x_1 = 8$, $x_2 = 5$, $x_3 = -5$, $x_4 = -4$, $y_1 = 6$, $y_2 = 11$, $y_3 = 12$ **and** $y_4 = 3$

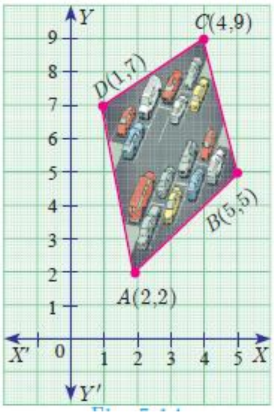
Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 11 & 12 & 3 & 6 \end{Bmatrix}$

$= \frac{1}{2}[(88+60-15-24) - (30-55-48+24)]$

$= \frac{1}{2}[(109) - (-49)]$ $= \frac{1}{2}(109+49)$ $= \frac{1}{2}(158)$ $= 79$

Therefore area of quadrilateral = 79 sq. units.

14. ***The given diagram shows a plan for constructing a new Parking loot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot ?***



Answer:

The parking lot is a quadrilateral whose vertices are at

A(2,2), B(5,5), C(4,9) and D(1,7)

$x_1 = 2$, $x_2 = 5$, $x_3 = 4$, $x_4 = 1$, $y_1 = 2$, $y_2 = 5$, $y_3 = 9$ **and** $y_4 = 7$

Area of the triangle $\Rightarrow \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{Bmatrix}$

$= \frac{1}{2}[\,(10 + 45 + 28 + 2\,) - (10 + 20 + 9 + 14\,)\,]$

$= \frac{1}{2}[\,(85) - (53)]$ $\quad = \frac{1}{2}(85 - 53\,)$ $\quad = \frac{1}{2}(\,32)$ $\quad = 16$

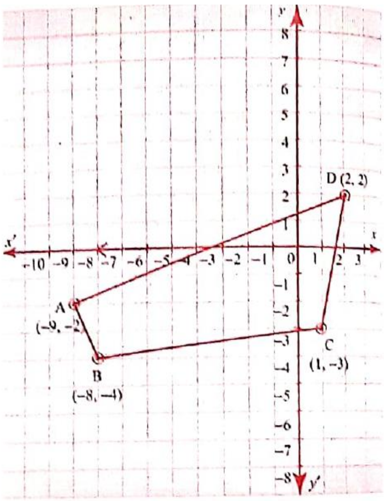
Therefore area of parking lot = 16 sq. feets.

Construction rate per square feet ₹1300.

Therefore total cost for constructing the parking lot = 16 x 1300 = ₹20800

15. ***Find the area of the quadrilateral whose verticies are at***
(i) (-9, -2), (-8,-4), (2,2) and (1,-3) (ii) (-9,0), (-8,6), (-1,-2) and (-6,-3)

(i) (-9, -2), (-8,-4), (2,2) and (1,-3)



A (-9, -2), B (-8, -4), C (1, -3) and D (2, 2)

x_1 = -9, x_2 = -8, x_3 = 1, x_4 = 2, y_1 = -2, y_2 = -4, y_3 = -3 and y_4 = 2

Area of the triangle $\Rightarrow \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{Bmatrix}$

$= \frac{1}{2}[\,(36 + 24 + 2 - 4\ \ ) - (16 - 4 - 6 - 18\,)]$

$= \frac{1}{2}[\,(58) - (-12)]$ $\quad = \frac{1}{2}(58 + 12\,)$

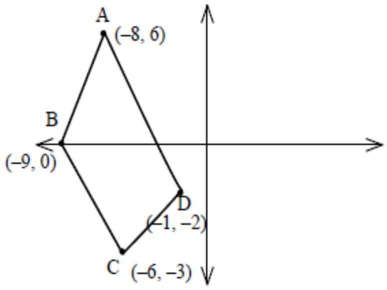
$= \frac{1}{2}(\,70)$ $\quad = 35$

Therefore area of quadrilateral = 35 sq. units.

(ii) (-9,0), (-8,6), (-1,-2) and (-6,-3)

A (-8, 6), B (-9, 0), C (-6, -3) and D (-1, -2)

x_1 = -8, x_2 = -9, x_3 = -6, x_4 = -1, y_1 = 6, y_2 = 0, y_3 = -3 and y_4 = -2



Area of the triangle $\Rightarrow \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{Bmatrix}$

$= \frac{1}{2}[\,(0 + 27 + 12 - 6\,) - (-54 + 0 + 3 + 16\,)]$

$= \frac{1}{2}[\,(33) - (-35)]$ $\quad = \frac{1}{2}(33 + 35\,)$

$= \frac{1}{2}(\,88)$ $\quad = 44$

Therefore area of quadrilateral = 44 sq. units.

16. ***Find the value of 'k' if the area of quadrilateral is 28 sq. units, whose vertices are (-4,-2), (-3,k), (3, -2) and (2, 3)***

Answer:

Area of the triangle $\Rightarrow \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

x_1 = -4, x_2 = -3, x_3 = 3, x_4 = 2, y_1 = -2, y_2 = k, y_3 = -2 and y_4 = 3

Area of quadrilateral $\Rightarrow \frac{1}{2}\left\{\begin{matrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{matrix}\right\} = 28$ *sq. units.*

$\Rightarrow \frac{1}{2}[\,(-4k + 6 + 9 - 4\,) - (6 + 3k - 4 - 12\,)] = 28$ *sq. units*

$\Rightarrow \frac{1}{2}[\,(\,11 - 4k) - (3k - 10)] \quad = 28$

$\Rightarrow \frac{1}{2}(11 - 4k - 3k + 10\,) \quad = 28$

$\Rightarrow 21 - 7k = 28 \text{ x } 2 \qquad \Rightarrow 21 - 7k = 56 \qquad \Rightarrow - 7k = 56 - 21$

$\Rightarrow - 7k = 35 \qquad \Rightarrow k = \frac{35}{-7}$

Therefore k = -5

17. ***If the points A (-3, 9), B (a, b) and C (4, -5) are collinear and if a + b = 1, then find 'a' and 'b'.***

Answer:

$x_1 = -3, \quad x_2 = a, \quad x_3 = 4, \quad y_1 = 9, \quad y_2 = b \quad \text{and} \quad y_3 = -5$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\} = 0$ *Sq. Units*

$\Rightarrow \frac{1}{2}\begin{pmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{pmatrix} = 0$

$\Rightarrow \frac{1}{2}[\,(-3b - 5a + 36\,) - (9a + 4b + 15\,)] = 0$

$\Rightarrow \frac{1}{2}[\,(-3b - 5a + 36 - 9a - 4b - 15\,] = 0 \qquad \Rightarrow \frac{1}{2}[-14a - 7b + 21] = 0$

$\Rightarrow -14a - 7b + 21 = 0 \text{ x } 2 \qquad \Rightarrow -14a - 7b + 21 = 0$

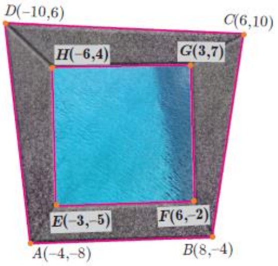
$\Rightarrow -14a - 7b = -21$ *(divided by – 7)*

$\Rightarrow 2a + b = 3$ --------------------(1)

$\Rightarrow a + b = 1$ --------------------(2) *(given)*

By solving (1) and (2) we get a = 2 and c = – 1

18. ***In the figure, the quadrilateral swimming pool shown is surrounded by the concrete patio. Find the area of the patio.***



Answer:

Required area of the patio = area of portion ABCD – Area of portion EFGH

Area of portion ABCD ***A (-4, -8), B (8, -4), C = (6, 10) and D (-10, 6)***

$x_1 = -4, \quad x_2 = 8, \quad x_3 = 6, \quad x_4 = -10, \quad y_1 = -8, y_2 = -4, \quad y_3 = 10 \quad \text{and} \quad y_4 = 6$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$ *Sq. Units*

Area of quadrilateral $= \frac{1}{2}\left\{\begin{matrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{matrix}\right\}$

$= \frac{1}{2}[\,(16 + 80 + 36 + 80\,) - (-64 - 24 - 100 - 24\,)]$

$= \frac{1}{2}[\,(\,212) - (\,-212\,)] = \frac{1}{2}(212 + 212\,)$

$= \frac{1}{2}(\,424) \qquad = 212$

Therefore area of quadrilateral ABCD = 212 sq. units.

Area of portion EFGH ***E (-3, - 5), F (6, -2), G = (3, 7) and H (-6, 4)***

$x_1 = -3, \quad x_2 = 6, \quad x_3 = 3, \quad x_4 = -6, \quad y_1 = -5, y_2 = -2, \quad y_3 = 7 \quad \text{and} \quad y_4 = 4$

Area of the triangle $\Rightarrow \frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right\}$ *Sq. Units*

Area of quadrilateral $=\frac{1}{2}\left\{\begin{matrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{matrix}\right\}$

$=\frac{1}{2}[\ 6 + 42 + 12 + 30\ ) - (-30 - 6 - 42 - 12)]$

$=\frac{1}{2}[\ (90) - (-90\ )]$ $\quad=\frac{1}{2}(90 + 90\ )$

$=\frac{1}{2}(\ 180)$ $\quad= 90$

Therefore area of quadrilateral ABCD = 90 sq. units.

Required area of the patio = area of portion ABCD – Area of portion EFGH

= 212 – 90

= 122 Sq. Units.

19. ***A triangular shaped glass with vertices at A (-5, -4), B (1, 6) and C (7, -4) has to bepainted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.***

Answer:

Area of the triangle $=\frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ *Sq. units.*

$x_1 = -5, \quad x_2 = 1, \quad x_3 = 7, \quad y_1 = -4, \quad y_2 = 6$ **and** $y_3 = -4$

Area of the triangle $=\frac{1}{2}\left(\begin{matrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{matrix}\right)$ $\quad=\frac{1}{2}[\ (-30 - 4\ - 28\ ) - (-4 + 42 + 20\ )]$

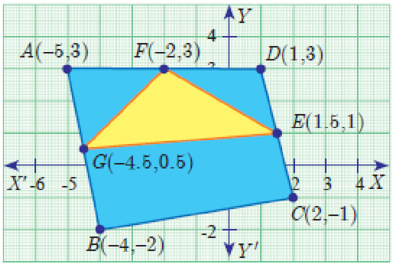
$=\frac{1}{2}[\ (-54\ ) - (\ 18)]$ $\quad=\frac{1}{2}(\ -62 - 58\ )$ $\quad=\frac{1}{2}(-120) = -60$

Area of the triangle = 60 Sq. Units. (Area can't be – ve)

Number of paint cans required $=\dfrac{Area\ of\ the\ \Delta\ given}{Area\ of\ the\ paint\ can} = \dfrac{60}{6}$

Number of paint cans required = 10 cans

20. ***In the figure, find area of (i) triangle AGF (ii) triangle FED (iii) Quadrilateral BCEG***



Answer:

(i) triangle AGF

A ((-5, 3), G (-4.5, 0.5) and F (-2, 3)

Area of the triangle $=\frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ *Sq. units.*

$x_1 = -5, \quad x_2 = -4.5, \quad x_3 = -2, \quad y_1 = 3, \quad y_2 = 0.5$ **and** $y_3 = 3$

Area of the triangle $=\frac{1}{2}\left(\begin{matrix} -5 & -4.5 & -2 & -5 \\ 3 & 0.5 & 3 & 3 \end{matrix}\right)$ $\quad=\frac{1}{2}[\ (-2.5 - 13.5 - 6\ ) - (-13.5 - 1 - 15\ )]$

$=\frac{1}{2}[\ (-22\ ) - (\ -29.5\ )]$ $\quad=\frac{1}{2}(\ -22 + 29.5\ )$ $\quad=\frac{1}{2}(\ 7.5) = 3.75$

Area of the triangle AGF = 3.75 Sq. Units.

(ii) triangle FED

F ((-2, 3), E (1.5, 1) and D (1, 3)

Area of the triangle $=\frac{1}{2}\left\{\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right\}$ *Sq. units.*

$x_1 = -2, \quad x_2 = 1.5, \quad x_3 = 1, \quad y_1 = 3, \quad y_2 = 1$ **and** $y_3 = 3$

Area of the triangle $=\frac{1}{2}\left(\begin{matrix} -2 & 1.5 & 1 & -2 \\ 3 & 1 & 3 & 3 \end{matrix}\right)$ $\quad=\frac{1}{2}[\ (-2 + 4.5 + 3\ ) - (\ 4.5 + 1 - 6\ )]$

$=\frac{1}{2}[\ (6.5\ ) - (\ -0.5\ )]$ $\quad=\frac{1}{2}(\ 5.5 + .5\ )$ $\quad=\frac{1}{2}(\ 6\ ) = 3$

Area of the triangle FED = 3 Sq. Units.

(iii) Quadrilateral BCEG

B (– 4, –2), C (2,– 1), E (1.5, 1) and G (– 4.5, 0.5)

x_1 = -4, x_2 = 2, x_3 = 1.5, x_4 = -4.5, y_1 = 2, y_2 = -1, y_3 = 1 and y_4 = 0.5

Area of the triangle $\Rightarrow \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$ *Sq. Units*

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} -4 & 2 & 1.5 & -4.5 & -4 \\ -2 & -1 & 1 & 0.5 & -2 \end{Bmatrix}$

$= \frac{1}{2}[\ 4 + 2 + 0.75 + 9\ ) - (- 4 - 1.5 - 4.5 - 2)]$

$= \frac{1}{2}[\ (15.75) - (- 4)] = \frac{1}{2}(15.\ 75 + 12\ )$

$= \frac{1}{2}(27.75)$ $= 13.875$

Therefore area of quadrilateral BCEG = 13.86 sq. units.

MATRIX

1. Consider the following information regaarding the number of men and women in three facatories, I, II and III

Factory	Men	Women
I	23	18
II	47	36
II	15	16

Represent the above information in the form of matrix. What does the entry in the second row and first column resresent?

Answer:

The information is represented in the form of a 3 x 2 matrix as follows

$A = \begin{bmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{bmatrix}$ *The entry in the second row and first column represent that there are 47 men workers in factory II*

2. If a matrix has 16 elements, what are the possible orders it can have?

Answer:

We know that a matrix of order m x n, has mn elements. Thus to find all possible ordoers of a matrix with 16 elements, we will find all ordered paris of natural numbers whose product is 16.

Such ordered pairs are (1, 16), (16, 1), (4, 4), (8, 2) and (2, 8)

Hence the possible orders are 1 x 16, 16 x 1, 4 x 4, 8 x 2, 2 x 8

3. Construct 3 x 3 matrix whose elements are $a_{ij} = i^2 j^2$

Answer:

The general 3 x 3 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $\Rightarrow$ **$a_{ij} = i^2 j^2$**

$a_{11} = 1^2 \times 1^2 = 1 \times 1 = 1$

$a_{12} = 1^2 \times 2^2 = 1 \times 4 = 4$

$a_{13} = 1^2 \times 3^2 = 1 \times 9 = 9$

$a_{21} = 2^2 \times 1^2 = 4 \times 1 = 4$

$a_{22} = 2^2 \times 2^2 = 4 \times 4 = 16$

$a_{23} = 2^2 \times 3^2 = 4 \times 9 = 36$

$a_{31} = 3^2 \times 1^2 = 9 \times 1 = 9$

$a_{32} = 3^2 \times 2^2 = 9 \times 4 = 36$

$a_{33} = 3^2 \times 3^2 = 9 \times 9 = 81$

Hence required matrix $\boldsymbol{A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}}$

4. ***Find the value of a, b, c, d from the equation*** $\boldsymbol{\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}}$

Answer:

The given matrices are equal. Thus all corresponding elements are equal.

Therefore, $a - b = 1$ -------------- (1)
$2a + c = 5$ --------------(2)
$2a - b = 0$---------------(3)
$3c + d = 2$ ---------------(4)

(3) gives $2a - b = 0$ $\Rightarrow 2a = b$ ---------------(5)
Put $2a = b$ *in equation (1)* $\Rightarrow a - 2a = 1$ $\Rightarrow -a = 1$ $\Rightarrow a = -1$
Put $a = -1$ *in equation (5)* $\Rightarrow 2(-1) = b$ $\Rightarrow -2 = b$ $\Rightarrow b = -2$
Put $a = -1$ *in equation (2)* $\Rightarrow 2\,(-1) + c = 5$ $\Rightarrow -2 + c = 5$ $\Rightarrow c = 5 + 2$ $\Rightarrow c = 7$
Put $c = 7$ *in equation (4)* $\Rightarrow 3\,(7) + d = 2$ $\Rightarrow 21 + d = 2$ $\Rightarrow d = 2 - 21$ $\Rightarrow d = -19$

Therefore a = -1, b = -2, c = 7 and d = -19

5. ***If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements ?***

Answer:

Given a matrix has 18 elements.
The possible orders of the matrix are 18 x 1, 1 x 18, 9 x 2, 2 x 9, 6 x 3 and 3 x 6.
If the matrix has 6 elements
The order are 1 x 6, 6 x 1, 3 x 2, 2 x 3

6. ***Construct a 3 x 3 matrix whose elements are given by*** $\boldsymbol{a_{ij} = |\, i - 2j\,|}$

Answer:

The general 3 x 3 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $\Rightarrow \boldsymbol{a_{ij} = |\, i - 2j\,|}$

$a_{11} = |\,1 - 2(1)| = |1 - 2\,| = |-1| = 1$
$a_{12} = |\,1 - 2(2)| = |1 - 4\,| = |-3| = 3$
$a_{13} = |\,1 - 2(3)| = |1 - 6\,| = |-5| = 5$
$a_{21} = |\,2 - 2(1)| = |2 - 2\,| = |0| = 0$
$a_{22} = |2 - 2(2)| = |2 - 4\,| = |-2| = 2$
$a_{23} = |\,2 - 2(3)| = |2 - 6\,| = |-4| = 4$
$a_{31} = |\,3 - 2(1)| = |3 - 2\,| = |1| = 1$
$a_{32} = |\,3 - 2(2)| = |3 - 4\,| = |-1| = 1$
$a_{33} = |\,3 - 2(3)| = |3 - 6\,| = |-3| = 3$

Hence required matrix $\boldsymbol{A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}}$

7. ***If*** $\boldsymbol{A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}}$ ***then find the transpose of A***

Answer:

$$A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix} \quad \Rightarrow \quad A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

8. ***If A =*** $\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ ***then find the transpose of – A***

Answer:

A = $\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ $\Rightarrow$ *- A =* $\begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$ $\Rightarrow$ $-A^T =$ $\begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

9. ***If A =*** $\begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ ***then verify*** $(A^T)^T = A$

Answer:

A = $\begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ $\Rightarrow A^T =$ $\begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$ $\Rightarrow (A^T)^T =$ $\begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ *= A*

10. ***Find the values of x, y and z from the following equations.***

(i) $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$ ***(ii)*** $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$ ***(iii)*** $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

(i) $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

Answer:

X = 3, y = 12 and z = 3

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

Answer:

$5 + z = 5$ $\Rightarrow z = 5 - 5$ $\boldsymbol{\Rightarrow z = 0}$
$x + y = 6$ $\Rightarrow x = 6 - y$ ------(1)
$xy = 8$ $\Rightarrow (6 - y)y = 8$ (by (1))
$6y - y^2 = 8$ $\Rightarrow 6y - y^2 - 8 = 0$ $\Rightarrow -y^2 + 6y - 8 = 0$
$\Rightarrow y^2 - 6y + 8 = 0$ $\Rightarrow (y - 4)(y - 2) = 0$
$\boldsymbol{\Rightarrow y = 4 \text{ and } y = 2}$
If $y = 4$ $\Rightarrow x = 6 - 4$ $\boldsymbol{\Rightarrow x = 2}$
If $y = 2$ $\Rightarrow x = 6 - 2$ $\boldsymbol{\Rightarrow x = 4}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Answer:

$x + y + z = 9$ -------------------(1)
$x + z = 5$ -------------------(2)
$y + z = 7$ -------------------(3)
Substitute equation (2) in (1)
we get $(x + z + y) = 9$ $\Rightarrow 5 + y = 9$ $\Rightarrow y = 9 - 5$ $\Rightarrow y = 4$

Substitute equation (3) in (1)

we get $(x + y + z) = 9$ $\Rightarrow x + 7 = 9$ $\Rightarrow x = 9 - 7$ $\Rightarrow x = 2$

Substitute 'x' and 'y' values in equation (1)

we get $(x + y + z) = 9$ $\Rightarrow 2 + 4 + z = 9$ $\Rightarrow 6 + z = 9$ $\Rightarrow z = 9 - 6$ $\Rightarrow z = 3$

Solution x = 2, y = 4 and z = 3

11. ***In the matrix A =*** $\begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$ ***write***

(i) The number of elements ***(ii) The order of the matrix***

(iii) Write the elements of $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$.

Answer:

(i) *A has 4 rows and 4 columns*

Therefore number of elements = 4 x 4 *= 16*

(ii) *Order of matrix = 4 x 4*

(iii) $a_{22} = \sqrt{7}$ $a_{23} = \frac{\sqrt{3}}{2}$ $a_{24} = 5$ $a_{34} = 0$ $a_{43} = -11$ $a_{44} = 1$

12. ***If A =*** $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ ***, B =*** $\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$***, find A + B***

Answer:

$$A + B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

13. ***Two examinations were conoducted for three groups of students namely group – 1, group – 2 , group – 3 and their data on average of marks for the subjects Tamil, English, Science and Mathematics are given below in the form of matrices A and B. Find the total marks of both the examinations for all the three groups.***

$$A = \begin{array}{c} \\ \textbf{group 1} \\ \textbf{group 2} \\ \textbf{group 3} \end{array} \begin{array}{c} \begin{array}{cccc} \textbf{Tamil} & \textbf{English} & \textbf{Science} & \textbf{Maths} \end{array} \\ \begin{pmatrix} 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{pmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \textbf{group 1} \\ \textbf{group 2} \\ \textbf{group 3} \end{array} \begin{array}{c} \begin{array}{cccc} \textbf{Tamil} & \textbf{English} & \textbf{Science} & \textbf{Maths} \end{array} \\ \begin{pmatrix} 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{pmatrix} \end{array}$$

Answer:

The total marks in both the examinations for all the three groups is the sum of the given matrices.

$$A + B = \begin{pmatrix} 22+20 & 15+38 & 14+15 & 23+40 \\ 50+18 & 62+12 & 21+17 & 30+80 \\ 53+81 & 80+47 & 32+52 & 40+18 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 42 & 53 & 29 & 63 \\ 68 & 74 & 38 & 110 \\ 134 & 127 & 84 & 58 \end{pmatrix}$$

14. ***If*** $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$, ***then find A + B***

Answer:

It is not possible to add A and B because they have different orders.

15. ***If*** $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ ***then find 2A + B***

Answer:

$$2A = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} \qquad \Rightarrow 2A = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix}$$

$$2A + B = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} \qquad \Rightarrow 2A + B = \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$2A + B = \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

16. ***If*** $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ ***then find 4A – 3B***

Answer:

$$4A = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} \qquad \Rightarrow 4A = \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix}$$

$$3B = 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix} \qquad \Rightarrow 3B = \begin{pmatrix} -21 & 12 & -9 \\ \frac{3}{4} & \frac{21}{2} & 9 \\ 15 & -18 & 27 \end{pmatrix}$$

$$4A - 3B = \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} - \begin{pmatrix} -21 & 12 & -9 \\ \frac{3}{4} & \frac{21}{2} & 9 \\ 15 & -18 & 27 \end{pmatrix}$$

$$4A - 3B = \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} - \begin{pmatrix} -21 & 12 & -9 \\ \frac{3}{4} & \frac{21}{2} & 9 \\ 15 & -18 & 27 \end{pmatrix} = \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} + \begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}$$

$$4A - 3B = \begin{pmatrix} 20+21 & 16-12 & -8+9 \\ 2-\frac{3}{4} & 3-\frac{21}{2} & 4\sqrt{2}-9 \\ 4-15 & 36+18 & 16-27 \end{pmatrix}$$

$$\mathbf{4A - 3B} = \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

17. ***Find the value of a, b, c, d, x, y from the following matrix equation***

$$\begin{pmatrix}\boldsymbol{d} & \mathbf{8}\\ \mathbf{3}\boldsymbol{b} & \boldsymbol{a}\end{pmatrix} + \begin{pmatrix}\mathbf{3} & \boldsymbol{a}\\ \mathbf{-2} & \mathbf{-4}\end{pmatrix} = \begin{pmatrix}\mathbf{2} & \mathbf{2}\boldsymbol{a}\\ \boldsymbol{b} & \mathbf{4}\boldsymbol{c}\end{pmatrix} + \begin{pmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{-5} & \mathbf{0}\end{pmatrix}$$

Answer:

First, we add the two matrices on both left and right hand sides we get

$$\begin{pmatrix}d+3 & 8+a\\ 3b-2 & a-4\end{pmatrix} = \begin{pmatrix}2+0 & 2a+1\\ b-5 & 4c+0\end{pmatrix} \quad \Rightarrow \begin{pmatrix}d+3 & 8+a\\ 3b-2 & a-4\end{pmatrix} = \begin{pmatrix}2 & 2a+1\\ b-5 & 4c+0\end{pmatrix}$$

Equating the corroesponding elements of the two matrices, we have

$d + 3 = 2 \quad \Rightarrow d = 2 - 3 \quad \Rightarrow d = -1$

$8 + a = 2a + 1 \quad \Rightarrow a - 2a = 1 - 8 \quad \Rightarrow -a = -7 \quad \Rightarrow a = 7$

$3b - 2 = b - 5 \quad \Rightarrow 3b - b = -5 + 2 \quad \Rightarrow 2b = -3 \quad \Rightarrow b = \frac{-3}{2}$

Substituting $a = 7$ *in* $a - 4 = 4c \quad \Rightarrow 7 - 4 = 4c \quad \Rightarrow 3 = 4c \quad c = \frac{3}{4}$

Therefore $\boldsymbol{a = 7,\ b = \frac{-3}{2},\ c = \frac{3}{4}}$ ***and*** $\boldsymbol{d = -1}$

18. ***If*** $\boldsymbol{A = \begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix},\ B = \begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix},\ C = \begin{pmatrix}5 & 3 & 0\\ -1 & -7 & 2\\ 1 & 4 & 3\end{pmatrix}}$

Compute the following (i) 3A + 2B – C ***(ii)*** $\boldsymbol{\frac{1}{2}A - \frac{3}{2}B}$

(i) 3A + 2B – C

$$3A = 3\begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix} \qquad \Rightarrow 3A = \begin{pmatrix}3 & 24 & 9\\ 9 & 15 & 0\\ 24 & 21 & 18\end{pmatrix}$$

$$2B = 2\begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix} \Rightarrow 2B = \begin{pmatrix}16 & -12 & -8\\ 4 & 22 & -6\\ 0 & 2 & 10\end{pmatrix}$$

$$3A + 2B - C = \begin{pmatrix}3 & 24 & 9\\ 9 & 15 & 0\\ 24 & 21 & 18\end{pmatrix} + \begin{pmatrix}16 & -12 & -8\\ 4 & 22 & -6\\ 0 & 2 & 10\end{pmatrix} - \begin{pmatrix}5 & 3 & 0\\ -1 & -7 & 2\\ 1 & 4 & 3\end{pmatrix}$$

$$3A + 2B - C = \begin{pmatrix}3 & 24 & 9\\ 9 & 15 & 0\\ 24 & 21 & 18\end{pmatrix} + \begin{pmatrix}16 & -12 & -8\\ 4 & 22 & -6\\ 0 & 2 & 10\end{pmatrix} + \begin{pmatrix}-5 & -3 & 0\\ 1 & 7 & -2\\ -1 & -4 & -3\end{pmatrix}$$

$$3A + 2B - C = \begin{pmatrix}3+16-5 & 24-12-3 & 9-8+0\\ 9+4+1 & 15+22+7 & 0-6-2\\ 24+0-1 & 21+2-4 & 18+10-3\end{pmatrix}$$

$$3A + 2B - C = \begin{pmatrix}14 & 9 & 1\\ 14 & 44 & -8\\ 23 & 19 & 25\end{pmatrix}$$

19. ***If*** $\boldsymbol{A = \begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix},\ B = \begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}}$ ***then verify that***

(i) A + B = B + A ***(ii) A + (-A) = (-A) + A = 0***

Answer:

(i) A + B = B + A

$LHS = A + B = \begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix} + \begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix} \Rightarrow A + B = \begin{pmatrix}1+5 & 9+7\\ 3+3 & 4+3\\ 8+1 & -3+0\end{pmatrix} \Rightarrow A + B = \begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix}$------(1)

$RHS= B + A= \begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix} + \begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix} \Rightarrow B + A = \begin{pmatrix}5+1 & 7+9\\ 3+3 & 3+4\\ 1+8 & 0-3\end{pmatrix} \Rightarrow B + A = \begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix}$------(2)

From (1) and (2) LHS = RHS

(ii) A + (-A) = (-A) + A = 0

Answer:

A = $\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}$ and $-A = \begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}$

$A + (-A) = \begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix} + \begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix} \Rightarrow A + (-A) = \begin{pmatrix}1-1 & 9-9\\ 3-3 & 4-4\\ 8-8 & -3+3\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}$----------(1)

$(-A) + A = \begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix} + \begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix} \Rightarrow (-A) + A = \begin{pmatrix}-1+1 & -9+9\\ -3+3 & -4+4\\ -8+8 & 3-3\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}$----------(2)

From (1) and (2) LHS = RHS

20. ***If A =*** $\begin{pmatrix}\mathbf{4} & \mathbf{3} & \mathbf{1}\\ \mathbf{2} & \mathbf{3} & \mathbf{-8}\\ \mathbf{1} & \mathbf{0} & \mathbf{-4}\end{pmatrix}$, ***B =*** $\begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{4}\\ \mathbf{1} & \mathbf{9} & \mathbf{2}\\ \mathbf{-7} & \mathbf{1} & \mathbf{-1}\end{pmatrix}$, ***C =*** $\begin{pmatrix}\mathbf{8} & \mathbf{3} & \mathbf{4}\\ \mathbf{1} & \mathbf{-2} & \mathbf{3}\\ \mathbf{2} & \mathbf{4} & \mathbf{-1}\end{pmatrix}$ ***, then verify that***

A + (B + C) = (A + B) + C

Answer:

LHS = A + (B + C)

$B + C = \begin{pmatrix}2 & 3 & 4\\ 1 & 9 & 2\\ -7 & 1 & -1\end{pmatrix} + \begin{pmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{pmatrix} \Rightarrow B + C = \begin{pmatrix}2+8 & 3+3 & 4+4\\ 1+1 & 9-2 & 2+3\\ -7+2 & 1+4 & -1-1\end{pmatrix}$

$B + C = \begin{pmatrix}10 & 6 & 8\\ 2 & 7 & 5\\ -5 & 5 & -2\end{pmatrix}$

$A + (B + C) = \begin{pmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{pmatrix} + \begin{pmatrix}10 & 6 & 8\\ 2 & 7 & 5\\ -5 & 5 & -2\end{pmatrix} = \begin{pmatrix}4+10 & 3+6 & 1+8\\ 2+2 & 3+7 & -8+5\\ 1-5 & 0+5 & -4-2\end{pmatrix}$

A + (B + C) = $\begin{pmatrix}14 & 9 & 9\\ 4 & 10 & -3\\ -4 & 5 & -6\end{pmatrix}$-----------------(1)

RHS : (A + B) + C

$A + B = \begin{pmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{pmatrix} + \begin{pmatrix}2 & 3 & 4\\ 1 & 9 & 2\\ -7 & 1 & -1\end{pmatrix} = \begin{pmatrix}4+2 & 3+3 & 1+4\\ 2+1 & 3+9 & -8+2\\ 1-7 & 0+1 & -4-1\end{pmatrix}$

$A + B = \begin{pmatrix}6 & 6 & 5\\ 3 & 12 & -6\\ -6 & 1 & -5\end{pmatrix}$

$(A + B) + C = \begin{pmatrix}6 & 6 & 5\\ 3 & 12 & -6\\ -6 & 1 & -5\end{pmatrix} + \begin{pmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{pmatrix} = \begin{pmatrix}6+8 & 6+3 & 5+4\\ 3+1 & 12-2 & -6+3\\ -6+2 & 1+4 & -5-1\end{pmatrix}$

$(A + B) + C = \begin{pmatrix}14 & 9 & 9\\ 4 & 10 & -3\\ -4 & 5 & -6\end{pmatrix}$------------------(2)

From (1) and (2) LHS = RHS

21. ***Find X and Y if X + y =*** $\begin{pmatrix} \mathbf{7} & \mathbf{0} \\ \mathbf{3} & \mathbf{5} \end{pmatrix}$ ***and X – Y =*** $\begin{pmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} \end{pmatrix}$

Answer:

$$\text{Given} \quad X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} \qquad \text{..........} \; (1)$$

$$X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \qquad \text{..........} \; (2)$$

$$(1) + (2) \;\Rightarrow\; 2X = \begin{pmatrix} 10 & 0 \\ 3 & 9 \end{pmatrix}$$

$$\Rightarrow \; X = \begin{pmatrix} 5 & 0 \\ 3/2 & 9/2 \end{pmatrix}$$

$$(1) - (2) \;\Rightarrow\; 2Y = \begin{pmatrix} 4 & 0 \\ 3 & 1 \end{pmatrix}$$

$$\Rightarrow \; Y = \begin{pmatrix} 2 & 0 \\ 3/2 & 1/2 \end{pmatrix}$$

22. ***If A =*** $\begin{pmatrix} \mathbf{0} & \mathbf{4} & \mathbf{9} \\ \mathbf{8} & \mathbf{3} & \mathbf{7} \end{pmatrix}$ ***, and B =*** $\begin{pmatrix} \mathbf{7} & \mathbf{3} & \mathbf{8} \\ \mathbf{1} & \mathbf{4} & \mathbf{9} \end{pmatrix}$ ***then find the value of (i) B – 5A (ii) 3A – 9B***

Answer:

(i) B – 5A

$$5A = 5\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} \qquad 5A = \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix}$$

$$B - 5A = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix} \quad \Rightarrow B - 5A = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} + \begin{pmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{pmatrix}$$

$$B - 5A = \begin{pmatrix} 7 + 0 & 3 - 20 & 8 - 45 \\ 1 - 40 & 4 - 15 & 9 - 35 \end{pmatrix}$$

$$\mathbf{B - 5A} = \begin{pmatrix} \mathbf{7} & \mathbf{-17} & \mathbf{-37} \\ \mathbf{-39} & \mathbf{-11} & \mathbf{-26} \end{pmatrix}$$

(ii) 3A – 9B

$$3A = 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} \qquad \Rightarrow 3A = \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix}$$

$$9B = 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} \qquad \Rightarrow 9B = \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$3A - 9B = \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} - \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix} \Rightarrow 3A - 9B = \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} + \begin{pmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{pmatrix}$$

$$3A - 9B = \begin{pmatrix} 0 - 63 & 12 - 27 & 27 - 72 \\ 24 - 9 & 9 - 36 & 21 - 81 \end{pmatrix}$$

$$3A - 9B = \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

23. ***Find the values of x, y, z if (i)*** $\begin{pmatrix} \boldsymbol{x - 3} & \boldsymbol{3x - z} \\ \boldsymbol{x + y + 7} & \boldsymbol{x + y + z} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{6} \end{pmatrix}$

(ii) $\boldsymbol{(x \quad y - z \quad z + 3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)}$

Answer :

(i) $\begin{pmatrix} \boldsymbol{x - 3} & \boldsymbol{3x - z} \\ \boldsymbol{x + y + 7} & \boldsymbol{x + y + z} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{6} \end{pmatrix}$

Equating the corroesponding elements of the two matrices, we have

X – 3 = 1 $\quad \Rightarrow$ *x = 1 + 3* $\quad \Rightarrow$ *x = 4* ---------------(1)

$3x - z = 0 \quad \Rightarrow 3(4) - z = 0 \quad \Rightarrow 12 - z = 0 \quad \Rightarrow -z = 0 - 12 \quad \Rightarrow z = -12$ (by (1))

$x + y + 7 = 1 \quad \Rightarrow 4 + y + 7 = 1 \quad \Rightarrow 11 + y = 1 \quad \Rightarrow y = 1 - 11 \quad \Rightarrow y = -10$

x = 4, y = –10 and z = –12

(ii) $\begin{pmatrix} x & y-z & z+3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

$x + y = 4$ --------------(1)

$y - z = 4$ --------------(2)

$z + 3 + 3 = 16 \quad \Rightarrow z + 6 = 16 \quad \Rightarrow z = 16 - 6 \quad \Rightarrow z = 10$

Substitute 'z' value in equation (2)

$y - 10 = 4 \quad \Rightarrow y = 4 + 10 \quad \Rightarrow y = 14$

Substitute 'y' value in equation (1)

$x + 14 = 4 \quad \Rightarrow x = 4 - 14 \quad \Rightarrow x = -10$

x = – 10, y = 14 and z = 10

24. **Find x and y if** $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Answer:

$4x - 2y = 4$ (divided by 2)

$2x - y = 2$ ------------(1)

$-3x + 3y = 6$ (divided by 3)

$-x + y = 2$ -------------(2)

(1) $\Rightarrow \quad 2x - y = 2$

(2) $\Rightarrow \quad -x + y = 2$

Adding, $\quad x = 4$

Substitute 'x'value in equation (1) or (2)

$2(4) - y = 2 \quad \Rightarrow 8 - y = 2 \quad \Rightarrow -y = 2 - 8 \quad \Rightarrow -y = -6$

$y = 6$

x = 4 and y = 6

25. **Find the non – zero values of x satisfying the matrix equation**

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

Answer:

Given $\quad x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$

$$\begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix} + \begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix} \Rightarrow \begin{pmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

Equating the corroesponding elements of the two matrices, we have

$2x + 10x = 48 \quad \Rightarrow 12x = 48 \quad \Rightarrow x = \frac{48}{12} \quad \Rightarrow$ **x = 4**

26. **Solve for x, y :** $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}$

Answer:

Given $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$\Rightarrow \quad x^2 - 4x = 5 \quad | \quad y^2 - 2y = 8$

$\Rightarrow \quad x^2 - 4x - 5 = 0 \quad | \quad \Rightarrow y^2 - 2y - 8 = 0$

$\Rightarrow \quad (x - 5)(x + 1) = 0 \quad | \quad \Rightarrow (y - 4)(y + 2) = 0$

$\Rightarrow \quad x = 5, -1 \quad | \quad \Rightarrow \therefore y = 4, y = -2$

27. ***If A =*** $\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{5} \end{pmatrix}$***, B =*** $\begin{pmatrix} \mathbf{8} & \mathbf{3} & \mathbf{1} \\ \mathbf{2} & \mathbf{4} & \mathbf{1} \\ \mathbf{5} & \mathbf{3} & \mathbf{1} \end{pmatrix}$***, find AB and BA***

Answer:

AB = $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix}$

AB = $\begin{pmatrix} 12 & 1 & 3 \\ 51 & 28 & 9 \end{pmatrix}$

BA does not exist.

28. ***If A =*** $\begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} \end{pmatrix}$ ***, B =*** $\begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{3} \end{pmatrix}$***, find AB and BA. Check if AB = BA***

Answer:

AB = $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ *x* $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix} 4+1 & 0+3 \\ 2+3 & 0+9 \end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix} 5 & 3 \\ 5 & 9 \end{pmatrix}$ *------------(1)*

BA = $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ *x* $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ $\Rightarrow$ *BA =* $\begin{pmatrix} 4+0 & 2+0 \\ 2+3 & 1+9 \end{pmatrix}$ $\Rightarrow$ *BA =* $\begin{pmatrix} 4 & 2 \\ 5 & 10 \end{pmatrix}$ *-----------(2)*

From (1) and (2) $AB \neq BA$

29. ***If A =*** $\begin{pmatrix} \mathbf{2} & -\mathbf{2}\sqrt{\mathbf{2}} \\ \sqrt{\mathbf{2}} & \mathbf{2} \end{pmatrix}$***, and B =*** $\begin{pmatrix} \mathbf{2} & \mathbf{2}\sqrt{\mathbf{2}} \\ -\sqrt{\mathbf{2}} & \mathbf{2} \end{pmatrix}$ ***Show that A and B satisfy commutative property with respect to matrix multiplication***

Answer:

We have show that AB = BA

AB = $\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ *x* $\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix} 4+4 & 4\sqrt{2}-4\sqrt{2} \\ 4\sqrt{2}-4\sqrt{2} & 4+4 \end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ *------(1)*

BA = $\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$ *x* $\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ $\Rightarrow$ *BA =* $\begin{pmatrix} 4+4 & -4\sqrt{2}+4\sqrt{2} \\ -2\sqrt{2}+2\sqrt{2} & 4+4 \end{pmatrix}$ $\Rightarrow$ *BA =* $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ *------(2)*

From (1) and (2) AB = BA

30. ***Solve*** $\begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \mathbf{4} \\ \mathbf{5} \end{pmatrix}$

Answer:

Given $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 2x & + & 1y \\ x & + & 2y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Equating the corroesponding elements of the two matrices, we have

2x+y = 4 ------------------(1) *x + 2y = 5 ------------------------(2)*

$$\begin{array}{rrl} (1) - 2 \times (2) \text{ gives} & 2x + y = & 4 \\ & \underline{2x + 4y = \ \ 10} & (-) \\ & -3y = -6 & \text{gives } y = 2 \end{array}$$

Substitute 'y' value in equation (1)

2x + 2 = 4 $\Rightarrow$ *2x = 4 – 2* $\Rightarrow$ *2x = 2* $\Rightarrow$ *x = 1*

x = 1 and y = 2

31. ***If A =*** $\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ -\mathbf{1} & \mathbf{3} \end{pmatrix}$ ***, B =*** $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ -\mathbf{4} & \mathbf{2} \end{pmatrix}$ ***, and C =*** $\begin{pmatrix} -\mathbf{7} & \mathbf{6} \\ \mathbf{3} & \mathbf{2} \end{pmatrix}$ ***verify that A (B + C) = AB + AC***

Answer:

LHS = A (B + C)

B + C = $\begin{pmatrix}1&2\\-4&2\end{pmatrix}+\begin{pmatrix}-7&6\\3&2\end{pmatrix}$ $\Rightarrow$ *B + C =* $\begin{pmatrix}1-7&2+6\\-4+3&2+2\end{pmatrix}$ $\Rightarrow$ *B + C =* $\begin{pmatrix}-6&8\\-1&4\end{pmatrix}$

A (B + C) = $\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}-6&8\\-1&4\end{pmatrix}$ $\Rightarrow$ *A (B + C) =* $\begin{pmatrix}-6-1&8+4\\6-3&-8+12\end{pmatrix}$

A (B + C) = $\begin{pmatrix}-7&12\\3&4\end{pmatrix}$ -------------------- *(1)*

RHS = AB + AC

AB = $\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}1&2\\-4&2\end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix}1-4&2+2\\-1-12&-2+6\end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix}-3&4\\-13&4\end{pmatrix}$

AC = $\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}-7&6\\3&2\end{pmatrix}$ $\Rightarrow$ *AC =* $\begin{pmatrix}-7+3&6+2\\7+9&-6+6\end{pmatrix}$ $\Rightarrow$ *AC =* $\begin{pmatrix}-4&8\\16&0\end{pmatrix}$

AB + AC = $\begin{pmatrix}-3&4\\-13&4\end{pmatrix}+\begin{pmatrix}-4&8\\16&0\end{pmatrix}$ $\Rightarrow$ *AB + AC =* $\begin{pmatrix}-3-4&4+8\\-13+16&4+0\end{pmatrix}$

AB + AC = $\begin{pmatrix}-7&12\\3&4\end{pmatrix}$ -------------------- *(2)*

From (1) and (2) LHS = RHS

32. ***If A =*** $\begin{pmatrix}\mathbf{1}&\mathbf{2}&\mathbf{1}\\\mathbf{2}&\mathbf{-1}&\mathbf{1}\end{pmatrix}$***, and B =*** $\begin{pmatrix}\mathbf{2}&\mathbf{-1}\\\mathbf{-1}&\mathbf{4}\\\mathbf{0}&\mathbf{2}\end{pmatrix}$ ***then show that*** $(AB)^T = B^TA^T$

Answer:

LHS = $(AB)^T$

AB = $\begin{pmatrix}1&2&1\\2&-1&1\end{pmatrix}\times\begin{pmatrix}2&-1\\-1&4\\0&2\end{pmatrix}$ $\Rightarrow$ *AB* $\begin{pmatrix}2-2+0&-1+8+2\\4+1+0&-2-4+2\end{pmatrix}$ $\Rightarrow$ *AB =* $\begin{pmatrix}0&9\\5&-4\end{pmatrix}$

$(AB)^T=\begin{pmatrix}0&5\\9&-4\end{pmatrix}$ ------------------- *(1)*

RHS = B^TA^T

$B^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix}$ $A^T=\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}$

$B^TA^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix}\times\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}$ $\Rightarrow B^TA^T\begin{pmatrix}2-2+0&4+1+0\\-1+8+2&-2-4+2\end{pmatrix}$

$B^TA^T=\begin{pmatrix}0&9\\9&-4\end{pmatrix}$ -------------------------*(2)*

From (1) and (2) LHS = RHS

33. ***Find the order of the product matrix AB if***

	(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**
Orders of A	**3x3**	**4x3**	**4x2**	**4x5**	**1x1**
Orders of B	**3x3**	**3x2**	**2x2**	**5x1**	**1x3**

34. ***If A is of order p x q and B is of order q x r what is the order AB and BA ?***

Answer:

Given :

A is of order p x q

B is of order q x r

Therefore order of AB = (p x q) x (q x r) = p x r

Order of BA is not defined (Number of columns in B & number of rows in A are not equal)

35. ***A has 'a' rows and 'a + 3 ' columns. B has 'b' rows and '17 – b' columns, and if both products AB and BA exists, find 'a' and 'b' ?***

Answer:

Given

Order of A is $a \times (a + 3)$

Order of B is $b \times (17 - b)$ *Product AB exist.*

$\Rightarrow a + 3 = b$ *(Number of columns in A = Number of rows in B)*

$\Rightarrow a - b = -3$ ---------------------(1)

Product BA exist

$\Rightarrow 17 - b = a$ *(Number of columns in B = Number of rows in A)*

$\Rightarrow a + b = 17$ ---------------------(2)

Solving (1) and (2)

$2a = 14 \qquad \Rightarrow a = 7$

Substitute 'a' value in (1)

$\Rightarrow 7 - b = -3 \qquad \Rightarrow -b = -3 - 7 \qquad \Rightarrow -b = -10 \quad \Rightarrow b = 10$

Therefore a = 7 and b = 10

36. ***If*** $A = \begin{pmatrix} \mathbf{2} & \mathbf{5} \\ \mathbf{4} & \mathbf{3} \end{pmatrix}$, $B = \begin{pmatrix} \mathbf{1} & \mathbf{-3} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}$***, find AB and BA. Check if AB = BA***

Answer:

$AB = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix} \quad \Rightarrow AB = \begin{pmatrix} 2+10 & -6+25 \\ 4+6 & -12+15 \end{pmatrix} \quad \Rightarrow AB = \begin{pmatrix} 12 & 19 \\ 10 & 3 \end{pmatrix}$ ------------(1)

$BA = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix} \times \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} \quad \Rightarrow BA = \begin{pmatrix} 2-12 & 5-9 \\ 4+20 & 10+15 \end{pmatrix} \quad \Rightarrow AB = \begin{pmatrix} -10 & -4 \\ 24 & 25 \end{pmatrix}$ -----------(2)

From (1) and (2) $AB \neq BA$

37. ***If*** $A = \begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{5} & \mathbf{-1} \end{pmatrix}$, $B = \begin{pmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{5} & \mathbf{2} \end{pmatrix}$***, and*** $C = \begin{pmatrix} \mathbf{1} & \mathbf{3} & \mathbf{2} \\ \mathbf{-4} & \mathbf{1} & \mathbf{3} \end{pmatrix}$ ***verify that A (B + C) = AB + AC***

Answer:

LHS = A (B + C)

$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix} \qquad \Rightarrow B + C = \begin{pmatrix} 1+1 & -1+3 & 2+2 \\ 3-4 & 5+1 & 2+3 \end{pmatrix}$

$\Rightarrow B + C = \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$

$A(B + C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix} \qquad \Rightarrow A(B + C) = \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix}$

$A(B + C) = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix}$ -------------------- (1)

RHS = AB + AC

$AB = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} \qquad \Rightarrow AB = \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix} \quad \Rightarrow AB = \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix}$

$AC = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix} \qquad \Rightarrow AC = \begin{pmatrix} 1-12 & 3+3 & 2+9 \\ 5+4 & 15-1 & 10-3 \end{pmatrix} \quad \Rightarrow AC = \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix}$

$AB + AC = \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix} + \begin{pmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{pmatrix} \qquad \Rightarrow AB + AC = \begin{pmatrix} 10-11 & 14+6 & 8+11 \\ 2+9 & -10+14 & 8+7 \end{pmatrix}$

$AB + AC = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix}$ -------------------- (2)

From (1) and (2) LHS = RHS

38. Show that the matrices A = $\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ **, B =** $\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ **satisfy commutative property AB = BA**

Answer:

AB = $\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ x $\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ $\Rightarrow$ AB = $\begin{pmatrix}1-6 & -2+2\\ 3-3 & -6+1\end{pmatrix}$ $\Rightarrow$ AB = $\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$ ------------(1)

BA = $\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ x $\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ $\Rightarrow$ BA = $\begin{pmatrix}1-6 & 2-2\\ -3+3 & -6+1\end{pmatrix}$ $\Rightarrow$ BA = $\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$ -----------(2)

From (1) and (2) AB = BA

39. Let A = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ **, B =** $\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ **, C =** $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ **Show that (i) A(BC) = (AB)C**

(ii) (A – B)C = AC – BC (iii) (A – B $)^T$ **=** $A^T - B^T$

(i) A(BC) = (AB)C

Answer:

LHS = A(BC)

BC = $\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ x $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\Rightarrow$ BC = $\begin{pmatrix}8+0 & 0+0\\ 2+5 & 0+10\end{pmatrix}$ $\Rightarrow$ BC = $\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$

A(BC) = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}x\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$ $\Rightarrow$ A(BC) = $\begin{pmatrix}8+14 & 0+20\\ 8+21 & 0+30\end{pmatrix}$ $\Rightarrow$ A(BC) = $\begin{pmatrix}22 & 20\\ 29 & 30\end{pmatrix}$-----------(1)

RHS :

(AB)C

AB = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ x $\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ $\Rightarrow$ AB = $\begin{pmatrix}4+2 & 0+10\\ 4+3 & 0+15\end{pmatrix}$ $\Rightarrow$ AB = $\begin{pmatrix}6 & 10\\ 7 & 15\end{pmatrix}$

(AB)C = = $\begin{pmatrix}6 & 10\\ 7 & 15\end{pmatrix}$ x $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\Rightarrow$ (AB)C = $\begin{pmatrix}12+10 & 0+20\\ 14+15 & 0+30\end{pmatrix}$ $\Rightarrow$ (AB)C = $\begin{pmatrix}22 & 20\\ 29 & 30\end{pmatrix}$----------(2)

From (1) and (2) LHS = RHS

(ii) (A – B)C = AC – BC

Answer:

LHS = (A – B) C

A – B = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ – $\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ $\Rightarrow$ A – B = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ + $\begin{pmatrix}-4 & 0\\ -1 & -5\end{pmatrix}$ $\Rightarrow$ A – B = $\begin{pmatrix}1-4 & 2+0\\ 1-1 & 3-5\end{pmatrix}$

A – B = $\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}$

(A – B) C = $\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}$ x $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\Rightarrow$ (A – B) C = $\begin{pmatrix}-6+2 & 0+4\\ 0-2 & 0-4\end{pmatrix}$

(A-B)C = $\begin{pmatrix}-4 & 4\\ -2 & -4\end{pmatrix}$ -------------------(1)

RHS = AC – BC

AC = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$ x $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\Rightarrow$ AC = $\begin{pmatrix}2+2 & 0+4\\ 2+3 & 0+6\end{pmatrix}$ $\Rightarrow$ AC = $\begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix}$

BC =$\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$ x $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $\Rightarrow$ BC = $\begin{pmatrix}8+0 & 0+0\\ 2+5 & 0+10\end{pmatrix}$ $\Rightarrow$ BC = $\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$

AC – BC = $\begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix}$ – $\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$ $\Rightarrow$ AC – BC = $\begin{pmatrix}4 & 4\\ 5 & 6\end{pmatrix}$ + $\begin{pmatrix}-8 & 0\\ -7 & -10\end{pmatrix}$

AC – BC = $\begin{pmatrix}4-8 & 4+0\\ 5-7 & 6-10\end{pmatrix}$ $\Rightarrow$ AC – BC = $\begin{pmatrix}-4 & 4\\ -2 & -4\end{pmatrix}$ -------------------(1)

From (1) and (2) LHS = RHS

(iii) $(A-B)^T = A^T - B^T$

Answer:

LHS = $(A-B)^T$

$A - B = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix} - \begin{pmatrix}4 & 0\\1 & 5\end{pmatrix} \quad \Rightarrow A - B = \begin{pmatrix}1 & 2\\1 & 3\end{pmatrix} + \begin{pmatrix}-4 & 0\\-1 & -5\end{pmatrix} \quad \Rightarrow A - B = \begin{pmatrix}1-4 & 2+0\\1-1 & 3-5\end{pmatrix}$

$A - B = \begin{pmatrix}-3 & 2\\0 & -2\end{pmatrix}$

$(A-B)^T = \begin{pmatrix}-3 & 0\\2 & -2\end{pmatrix}$ ----------------*(1)*

$A^T = \begin{pmatrix}1 & 1\\2 & 3\end{pmatrix} \quad \Rightarrow B^T = \begin{pmatrix}4 & 1\\0 & 5\end{pmatrix}$

$A^T - B^T = \begin{pmatrix}1 & 1\\2 & 3\end{pmatrix} - \begin{pmatrix}4 & 1\\0 & 5\end{pmatrix} \quad \Rightarrow A^T - B^T = \begin{pmatrix}1 & 1\\2 & 3\end{pmatrix} + \begin{pmatrix}-4 & -1\\0 & -5\end{pmatrix} \quad \Rightarrow A^T - B^T = \begin{pmatrix}1-4 & 1-1\\2+0 & 3-5\end{pmatrix}$

$A^T - B^T = \begin{pmatrix}-3 & 0\\2 & -2\end{pmatrix}$ ----------------*(2)*

From (1) and (2) LHS = RHS

40. ***If A = $\begin{pmatrix}\cos\theta & 0\\0 & \cos\theta\end{pmatrix}$, B = $\begin{pmatrix}\sin\theta & 0\\0 & \sin\theta\end{pmatrix}$ Then show that $A^2 + B^2 = I$***

Answer:

$A^2 = \begin{pmatrix}\cos\theta & 0\\0 & \cos\theta\end{pmatrix} x \begin{pmatrix}\cos\theta & 0\\0 & \cos\theta\end{pmatrix} \quad \Rightarrow A^2 = \begin{pmatrix}\cos^2\theta + 0 & 0+0\\0+0 & 0+\cos^2\theta\end{pmatrix}$

$A^2 = \begin{pmatrix}\cos^2\theta & 0\\0 & \cos^2\theta\end{pmatrix}$

$B^2 = \begin{pmatrix}\sin\theta & 0\\0 & \sin\theta\end{pmatrix} X \begin{pmatrix}\sin\theta & 0\\0 & \sin\theta\end{pmatrix} \quad \Rightarrow B^2 = \begin{pmatrix}\sin^2\theta + 0 & 0+0\\0+0 & 0+\sin^2\theta\end{pmatrix}$

$B^2 = \begin{pmatrix}\sin^2\theta & 0\\0 & \sin^2\theta\end{pmatrix}$

$A^2 + B^2 = \begin{pmatrix}\cos^2\theta & 0\\0 & \cos^2\theta\end{pmatrix} + = \begin{pmatrix}\sin^2\theta & 0\\0 & \sin^2\theta\end{pmatrix} \quad \Rightarrow \begin{pmatrix}\cos^2\theta + \sin^2\theta & 0\\0 & \cos^2\theta + \sin^2\theta\end{pmatrix}$

$A^2 + B^2 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \quad \Rightarrow A^2 + B^2 = I$ ***($\sin^2\theta + \cos^2\theta = 1$)***

41. ***If A = $\begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix}$ prove that $AA^T = I$***

Answer:

$A = \begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix} \quad \Rightarrow A^T = \begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}$

$AA^T = \begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix} x \begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}$

$AA^T = \begin{pmatrix}\cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \sin\theta\cos\theta\\\sin\theta & \sin^2\theta + \cos^2\theta\end{pmatrix} \Rightarrow AA^T = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \quad \Rightarrow \boldsymbol{AA^T = I}$

42. ***Verify $A^2 = I$ when A = $\begin{pmatrix}5 & -4\\6 & -5\end{pmatrix}$***

Answer:

$A^2 = \begin{pmatrix}5 & -4\\6 & -5\end{pmatrix} x \begin{pmatrix}5 & -4\\6 & -5\end{pmatrix} \quad \Rightarrow A^2 = \begin{pmatrix}25-24 & -20+20\\30-30 & -24+25\end{pmatrix} \quad \Rightarrow A^2 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$A^2 = I$

43. ***If A = $\begin{pmatrix}a & b\\c & d\end{pmatrix}$ and $I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ show that $A^2 - (a+d)A = (bc - ad)I_2$***

Answer:

LHS = $A^2-(a+d)A$

$A^2 = A \times A \quad \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \Rightarrow A^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$ ---------------------(1)

$(a+d)A = (a+d) \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \Rightarrow (a+d)A = \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{pmatrix}$---------------------(2)

From (1) and (2) we get,

$$A^2-(a+d)A = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} - \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{pmatrix}$$

$$= \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} + \begin{pmatrix} -a^2-ad & -ab-bd \\ -ac-cd & -ad-d^2 \end{pmatrix}$$

$$= \begin{pmatrix} a^2+bc-a^2-ad & ab+bd-ab-bd \\ ac+cd-ac-cd & bc+d^2-ad-d^2 \end{pmatrix}$$

$$= \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix}$$

$$= (bc-ad)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= (bc-ad)\, I_2$$

Therefore LHS = RHS

44. **If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ then show that $(AB)^T = B^T A^T$**

Answer:

LHS = $(AB)^T$

$AB = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix} \times \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix} \qquad \Rightarrow AB = \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix} \quad \Rightarrow AB = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$ ---------------------(1)

RHS = $B^T A^T$

$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \qquad \Rightarrow A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix} \qquad \Rightarrow B^T A^T = \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$ ---------------------(2)

From (1) and (2) LHS = RHS

45. **If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7 I_2 = 0$**

Answer:

$A^2 = A \times A \qquad \Rightarrow A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \times \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} \quad \Rightarrow A^2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$

$5A = 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \qquad \Rightarrow 5A = \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix}$

$7\,I_2 = 7\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $\Rightarrow 7\,I_2 = \begin{pmatrix}7 & 0\\0 & 7\end{pmatrix}$

$A^2 - 5A + 7\,I_2 = \begin{pmatrix}8 & 5\\-5 & 3\end{pmatrix} - \begin{pmatrix}15 & 5\\-5 & 10\end{pmatrix} + \begin{pmatrix}7 & 0\\0 & 7\end{pmatrix}$

$A^2 - 5A + 7\,I_2 = \begin{pmatrix}8 & 5\\-5 & 3\end{pmatrix} + \begin{pmatrix}-15 & -5\\5 & -10\end{pmatrix} + \begin{pmatrix}7 & 0\\0 & 7\end{pmatrix}$

$A^2 - 5A + 7\,I_2 = \begin{pmatrix}8-15+7 & 5-5+0\\-5+5+0 & 3-10+7\end{pmatrix}$

$A^2 - 5A + 7\,I_2 = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ $\Rightarrow A^2 - 5A + 7\,I_2 = 0$

46. If $A = \begin{pmatrix}1 & -1 & 2\end{pmatrix}$, $B = \begin{pmatrix}1 & -1\\2 & 1\\1 & 3\end{pmatrix}$ and $C = \begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$ Show that (AB)C = A(BC)

Answer:

LHS = (AB)C

$AB = \begin{pmatrix}1 & -1 & 2\end{pmatrix} \times \begin{pmatrix}1 & -1\\2 & 1\\1 & 3\end{pmatrix}$ $\Rightarrow AB = \begin{pmatrix}1-2+2 & -1-1+6\end{pmatrix}$ $\Rightarrow AB = \begin{pmatrix}1 & 4\end{pmatrix}$

$(AB)C = \begin{pmatrix}1 & 4\end{pmatrix} \times \begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$ $\Rightarrow (AB)C = \begin{pmatrix}1+8 & 2-4\end{pmatrix}$

$\Rightarrow (AB)C = \begin{pmatrix}9 & -2\end{pmatrix}$--------------------------(1)

RHS= A(BC)

$BC = \begin{pmatrix}1 & -1\\2 & 1\\1 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$ $\Rightarrow BC = \begin{pmatrix}1-2 & 2+1\\2+2 & 4-1\\1+6 & 2-3\end{pmatrix}$ $\Rightarrow BC = \begin{pmatrix}-1 & 3\\4 & 3\\7 & -1\end{pmatrix}$

$A(BC) = \begin{pmatrix}1 & -1 & 2\end{pmatrix} \times \begin{pmatrix}-1 & 3\\4 & 3\\7 & -1\end{pmatrix}$ $\Rightarrow A(BC) = \begin{pmatrix}-1-4+14 & 3-3-2\end{pmatrix}$

$\Rightarrow A(BC) = \begin{pmatrix}9 & -2\end{pmatrix}$--------------------------(2)

From (1) and (2) LHS = RHS

SQUARE ROOT

1. ***Find the square root of the following expressions***

(i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$ ***(ii)*** $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Answer:

(i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

Answer:

$\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$ $= 16|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}|$

(ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Answer:

$$\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{12}{9}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right| = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

2. ***Find the square root of the following expressions.***

Answer:

Answer:

$\sqrt{16x^2+9y^2-24xy+24x-18y+9}$ $= \sqrt{(4x)^2+(-3y)^2+(3)^2+2(4x)(-3y)+2(-3y)(3)+2(4x)(3)}$

$= \sqrt{(4x-3y+3)^2}$

$= |4x-3y+3|$

(ii) $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$

Answer:

$\sqrt{(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)}$ $= \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$

$= \sqrt{(3x-1)^2(2x+1)^2(x+1)^2}$

$= |(3x-1)(2x+1)(x+1)|$

(iii) $[\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}][\sqrt{5}x^2+(2\sqrt{5}+1)x+2][\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}]$

Answer:

$\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}$ $= \sqrt{15}x^2+\sqrt{3}x+\sqrt{10}x+\sqrt{2}$

$= \sqrt{3}x(\sqrt{5}x+1)+\sqrt{2}(\sqrt{5}x+1)$

$= (\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})$

$\sqrt{5}x^2+(2\sqrt{5}+1)x+2$ $= \sqrt{5}x^2+2\sqrt{5}x+x+2$

$= \sqrt{5}x(x+2)+1(x+2)$

$= (x+2)(\sqrt{5}x+1)$

$\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}$ $= \sqrt{3}x^2+\sqrt{2}x+2\sqrt{3}x+2\sqrt{2}$

$= x(\sqrt{3}x+\sqrt{2})+2(\sqrt{3}x+\sqrt{2})$

$= (x+2)(\sqrt{3}x+\sqrt{2})$

$$\sqrt{[\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}][\sqrt{5}x^2+(2\sqrt{5}+1)x+2][\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}]}$$

= $\sqrt{(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(x+2)(\sqrt{5}x+1)(x\ +\ 2\ )(\sqrt{3}x+\sqrt{2})}$

= $\sqrt{(\sqrt{5}x+1)^2\left((\sqrt{3}x+\sqrt{2})\right)^2(x+2)^2}$

= $|(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(x+2)|$

3. ***Find the square root of the following rational expressions.***

(i) $\dfrac{400x^4y^{12}c^{16}}{100x^8y^4z^4}$ ***(ii)*** $\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$ ***(iii)*** $\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Answer:

(i) $\dfrac{400x^4y^{12}c^{16}}{100x^8y^4z^4}$

Answer

$\sqrt{\dfrac{400x^4y^{12}c^{16}}{100x^8y^4z^4}} = \dfrac{20|x^2y^6z^8|}{10|x^4y^2z^2|} = 2\left|\dfrac{y^4z^6}{x^2}\right|$

(ii) $\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$

Answer:

$$\sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}} = \sqrt{\frac{(\sqrt{7}x+2)(\sqrt{7}x+2)}{\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}\right)}}$$

$$= \frac{(\sqrt{7}x+2)}{\left(x-\frac{1}{4}\right)} = \frac{(\sqrt{7}x+2)}{\frac{4x-1}{4}}$$

$$= 4\left|\frac{(\sqrt{7}x+2)}{(4x-1)}\right|$$

(iii) $\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Answer:

$$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}} = \left|\frac{11\ (a+b)^4(x+y)^4(b-c)^4}{9(b-c)^2(a-b)^6(b-c)^2}\right|$$

$$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$$

4. ***Find the square root of the following:***

(i) $4x^2+20x+25$ ***(ii)*** $9x^2-24xy+30xz-40yz+25z^2+16y^2$

(iii) $1+\dfrac{1}{x^6}+\dfrac{2}{x^3}$ ***(iv)*** $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

(v) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Answer:

(i) $4x^2 + 20x + 25$

$\sqrt{4x^2+20x+25}$ = $\sqrt{(2x+5)^2}$ = $|2x+5|$

(ii) $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

$\sqrt{9x^2+16y^2+25z^2-24xy+30xz-40yz}$

= $\sqrt{(3x)^2+(-4y)^2+(5z)^2+2(4x)(-3y)+2(-4y)(5z)+2(5z)(3x)}$

= $\sqrt{(3x-4y+5z)^2}$

= $|3x-4y+5z|$

(iii) $1 + \frac{1}{x^6} + \frac{2}{x^3}$

$\sqrt{+\frac{1}{x^6}+\frac{2}{x^3}}$ = $\sqrt{1^2+2\,(1)\left(\frac{1}{x^3}\right)+\left(\frac{1}{x^3}\right)^2}$

= $\sqrt{\left(1+\frac{1}{x^3}\right)^2}$ = $\left|1+\frac{1}{x^3}\right|$

(iv) $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

$\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)}$

= $\sqrt{(x-2)(4x-1)(x-2)(7x+1)(4x-1)(7x+1)}$

= $\sqrt{(x-2)^2(4x-1)^2(7x+1)^2}$

= $|(x-2)(4x-1)(7x+1)|$

(v) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

$\sqrt{\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)}$

= $\sqrt{\left(\frac{12x^2+17x+6}{6}\right)\left(\frac{3x^2+8x+4}{2}\right)\left(\frac{4x^2+11x+6}{3}\right)}$

= $\frac{1}{6}\sqrt{(12x^2+17x+6)(3x^2+8x+4)(4x^2+11x+6)}$

= $\frac{1}{6}\sqrt{(4x+3)(3x+2)(x+2)(3x+2)(4x+3)(x+2)}$

= $\frac{1}{6}\sqrt{(4x+3)^2(3x+2)^2(x+2)^2}$

= $\frac{1}{6}\,|(4x+3)(3x+2)(x+2)|$

5. Find the square root of the following polynomials by division method.

(i) $x^4 - 12x^3 + 42x^2 - 36x + 9$

$$\begin{array}{l|l} & x^2 - 6x + 3 \\ \hline x^2 & x^4 - 12x^3 + 42x^2 - 36x + 9 \\ & x^4 \qquad (-) \\ \hline 2x^2 - 6x & -12x^3 + 42x^2 \\ & -12x^3 + 36x^2 \qquad (-) \\ \hline 2x^2 - 12x + 3 & +6x^2 - 36x + 9 \\ & +6x^2 - 36x + 9 \qquad (-) \\ \hline & 0 \end{array}$$

$\sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9}$ **= | $x^2 - 6x + 3$ |**

(ii) $37x^2 - 28x^3 + 4x^4 + 42x + 9$

$$\begin{array}{l|l} & 2x^2 - 7x - 3 \\ \hline 2x^2 & 4x^4 - 28x^3 + 37x^2 + 42x + 9 \\ & 4x^4 \qquad (-) \\ \hline 4x^2 - 7x & -28x^3 + 37x^2 \\ & -28x^3 + 49x^2 \qquad (-) \\ \hline 4x^2 - 14x - 3 & -12x^2 + 42x + 9 \\ & -12x^2 + 42x + 9 \qquad (-) \\ \hline & 0 \end{array}$$

$\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9}$ **= | $2x^2 - 7x - 3$ |**

(iii) $16x^4 + 8x^2 + 1$

$$\begin{array}{l|l} & 4x^2 - 0x + 1 \\ \hline 4\,x^2 & 16\,x^4 - 0x^3 + 8x^2 + 0x + 1 \\ & 16x^4 \qquad (-) \\ \hline 8x^2 - 0x & -0x^3 + 8x^2 \\ & -0x^3 + 0x^2 \qquad (-) \\ \hline 8x^2 - 0x + 1 & +8x^2 + 0x + 1 \\ & +8x^2 + 0x + 1 \qquad (-) \\ \hline & 0 \end{array}$$

$\sqrt{16x^4 - 0x^3 + 8x^2 + 0x + 1}$ **= | $4x^2 - 0x + 1$ |**

(iv) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

$$\begin{array}{l|l} & 11x^2 - 9x - 12 \\ \hline 11x^2 & 121x^4 - 198x^3 - 183x^2 - 216x + 144 \\ & 121x^4 \qquad (-) \\ \hline 22x^2 - 9x & -198x^3 - 183x^2 \\ & -198x^3 + 81x^2 \qquad (-) \\ \hline 22x^2 - 18x - 12 & -264x^2 - 216x + 144 \\ & -264x^2 - 216x + 144 \quad (-) \\ \hline & 0 \end{array}$$

$\sqrt{121x^4 - 198x^3 - 183x^2 - 216x + 144}$ **= | $11x^2 - 9x - 12$ |**

(v) $64x^4 - 16x^3 + 17x^2 - 2x + 1$

$$\begin{array}{l|ll}
 & 8x^2 - x + 1 & \\ \hline
8x^2 & 64x^4 - 16x^3 + 17x^2 - 2x + 1 & \\
 & 64x^4 & (-) \\ \hline
16x^2 - x & -16x^3 + 17x^2 & \\
 & -16x^3 + x^2 & (-) \\ \hline
16x^2 - 2x + 1 & 16x^2 - 2x + 1 & \\
 & 16x^2 - 2x + 1 & (-) \\ \hline
 & 0 &
\end{array}$$

$$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = | 8x^2 - x + 1 |$$

(vi) $\frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}$

$$\begin{array}{l|l}
 & \frac{x}{y} - 5 + \frac{y}{x} \\ \hline
\frac{x}{y} & \frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2} \\
 & \frac{x^2}{y^2} \\ \hline
\frac{2x}{y} - 5 & -10\frac{x}{y} + 27 \\
 & -10\frac{x}{y} + 25 \\ \hline
\frac{2x}{y} - 10 + \frac{y}{x} & 2 - 10\frac{y}{x} + \frac{y^2}{x^2} \\
 & 2 - 10\frac{y}{x} + \frac{y^2}{x^2} \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2}} = \left| \frac{x}{y} - 5 + \frac{y}{x} \right|$$

(vii) $\frac{4x^2}{y^2} + \frac{10x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

$$\begin{array}{l|ll}
 & \frac{2x}{y} + 5 - \frac{3y}{x} & \\ \hline
\frac{2x}{y} & \frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2} & \\
 & \frac{4x^2}{y^2} & (-) \\ \hline
\frac{4x}{y} + 5 & \frac{20x}{y} + 13 & \\
 & \frac{20x}{y} + 25 & (-) \\ \hline
\frac{4x}{y} + 10 - \frac{3y}{x} & -12 - \frac{30y}{x} + \frac{9y^2}{x^2} & \\
 & -12 - \frac{30y}{x} + \frac{9y^2}{x^2} & (-) \\ \hline
 & 0 &
\end{array}$$

Hence,

$$\sqrt{\frac{4x^2}{y} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}} = \left| \frac{2x}{y} + 5 - \frac{3y}{x} \right|$$

6. Find the values of 'a' and 'b' if the following polymnomials are perfect squares.

(i) $4x^4 - 12x^3 + 36x^2 + bx + a$

	$2x^2 - 3x + 7$	
$2x^2$	$\cancel{4x^4} - 12x^3 + 36x^2 + bx + a$	
	$\cancel{4x^4}$	(–)
$4x^2 - 3x$	$\cancel{-12x^3} + 36x^2$	
	$\cancel{-12x^3} + 9x^2$	(–)
$4x^2 - 6x + 7$	$28x^2 + bx + a$	
	$28x^2 - 42x + 49$	(–)
	0	

$a = 49$ and $b = -42$

(ii) $ax^4 + bx^3 + 361x^2 + 220x + 100$

	$10 + 11x + 12x^2$	
10	$\cancel{100} + 220x + 361x^2 + bx^3 + ax^4$	
	$\cancel{100}$	(–)
$20 + 11x$	$\cancel{+220x} + 361x^2$	
	$\cancel{220x} + 121x^2$	(–)
$20 + 22x + 12x^2$	$240x^2 + bx^3 + ax^4$	
	$240x^2 + 264x^3 + 144x^4$	(–)
	0	

$a = 144$ and $b = 264$

(iii) $x^4 - 8x^3 + ax^2 + bx + 16$

	$x^2 - 4x + 4$	
x^2	$\cancel{x^4} - 8x^3 + ax^2 + bx + 16$	
	$\cancel{x^4}$	(–)
$2x^2 - 4x$	$\cancel{-8x^3} + ax^2$	
	$\cancel{-8x^3} + 16x^2$	(–)
$2x^2 - 8x + 4$	$ax^2 - 16x^2 + bx + 16$	
	$8x^2 - 32x + 16$	(–)
	0	

$a - 16 = 8 \Rightarrow a = 8 + 16$

$\Rightarrow a = 24$ and $b = -32$

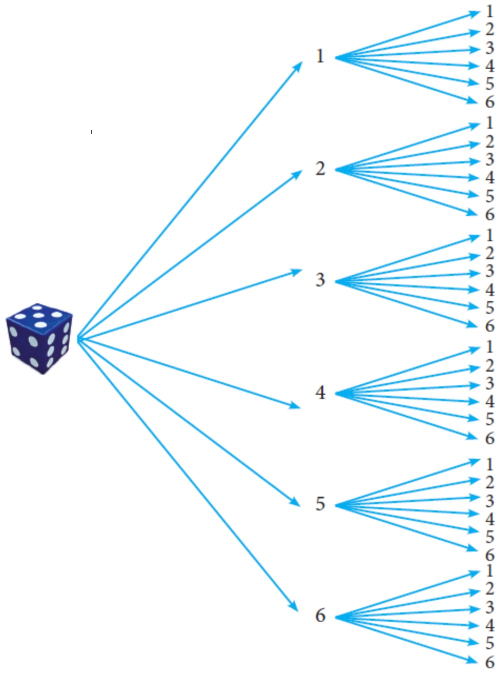
(iv) $\frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{a}{x} + b$

	$\frac{1}{x^2} - \frac{3}{x} + 2$	
$\frac{1}{x^2}$	$\cancel{\frac{1}{x^4}} - \frac{6}{x^4} + \frac{13}{x^2} + \frac{a}{x} + b$	
	$\cancel{\frac{1}{x^4}}$	(–)
$\frac{2}{x^2} - \frac{3}{x}$	$\cancel{-\frac{6}{x^4}} + \frac{13}{x^2}$	
	$\cancel{-\frac{6}{x^4}} + \frac{9}{x^2}$	(–)
$\frac{2}{x^2} - \frac{6}{x} + 2$	$\cancel{+\frac{4}{x^2}} + \frac{a}{x} + b$	
	$\cancel{+\frac{4}{x^2}} - \frac{12}{x} + 4$	(–)
	0	

$a = -12$ and $b = 4$

PROBABILITY

1. Express the sample space for rolling two dice using tree diagram.



S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36

2. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Answer:

Total number of possible outcomes (sample space) ⇒ n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball

Number of blue balls = 5 ⇒ n(A) = 5

Probability that the ball drawn is blue. Therefore

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

(ii) Let B will be the event of not getting a blue ball

n (S) = 4 (other than blue ball)

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{9}$$

3. Two dice are rolled. Find the probability that sum of outcomes is (i) equal to 4, (ii) greater than 10, (iii) less than 13

Answer:

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} ⇒ ***n(S) = 36***

(i) Let A be the event of getting the sum of outcome values equal to 4

A = { (1,3),(2,2), (3,1) } ⇒ *n(A) = 3*

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(ii) Let B be the event of getting the sum of outcome values greater than 10

B = { (5,6), (6,5), (6,6) } $\Rightarrow$ *n(B) = 3*

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(iii) Let C be the event op getting the sum of outcomes less than 13.

n(C) = n(S) $\Rightarrow$ *n(C) = 36*

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

4. ***Two coins are tossed together. What is the probability of getting different faces of the coins ?***

Answer:

When two coins are tossed together, the sample space is

S = { HH, HT, TH, TT } $\Rightarrow$ *n(S) = 4*

Let A be the event of getting different faces on the coins

A = { HT, TH} $\Rightarrow$ *n(A) = 2*

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

5. ***From a well shuffled a pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card.***

Answer:

n(S) = 52

(i) Let A be the event of getting a red card

n(A) = 13 + 13 $\Rightarrow$ *n(A) = 26*

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card,

n(B) = 13

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C be the event of getting a red king

n(C) = 1 + 1 $\Rightarrow$ *n(C) = 2*

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card. The face cards are Jack(J), Queen (Q) and King (K)

n(D) = 3 + 3 + 3 + 3 $\Rightarrow$ *n(D) = 12*

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9, 10.

n(E) = 9 + 9 + 9 + 9 $\Rightarrow$ *n(E) = 36*

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

6. ***What is the probability that leap year selected are random will contain 53 Saturdays.***

(Hint 366 = 52 x 7 + 2)

Answer:

A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

S = { (sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun)}

⇒ n(S) = 7

Let A be the event of getting 53 Saturdays in a leap year

A = { (Fri, Sat), (Sat, Sun)} ⇒ n(A) = 2

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

7. *A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows and odd number and the coin shows a head.*

Answer:

S = { 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T } ⇒ n(S) = 12

Let A be the event of getting an odd number and a head

A = { 1H, 3H, 5H } ⇒ n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

8. *A bag contains 6 green balls, some black and red balls. Number of black balls is twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.*

Answer:

Number of green balls is n(G) = 6

Let the number of red balls is n(R) = x

Therefore, number of black balls is n(B) = 2x

Total number of balls = 6 + x + 2x ⇒ n(S) = 6 + 3x

Given ⇒ P(G) = 3 P(R)

$$P(G) = 3\,P(R) \quad \Rightarrow \frac{n(g)}{n(S)} = 3\,\frac{n(R)}{n(S)}$$

$$\Rightarrow \frac{6}{6+3x} = 3\,\frac{x}{6+3x} \quad \Rightarrow 3x = 6 \quad \Rightarrow x = \frac{6}{3} \quad \Rightarrow x = 2$$

(i) Number of black balls = 2x ⇒ 2 x 2 = 4

(ii) Total number of balls = n(G) + n(R) + n(B) ⇒ 6 + 2 + 4

Total number of balls **⇒ n(S) = 12**

9. *A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, 4, ………………, 12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number ?*

Answer:

Sample space (S) = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 } ⇒ n(S) = 12

(i) Let A be the event that arrow will come to rest in 7

A = { 7} ⇒ n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event that arrow will come to rest in a prime number

B = { 2, 3, 5, 7, 11 } ⇒ n(B) = 5

$P(B) = \frac{n(B)}{n(S)} \quad = \frac{5}{12}$

(iii) Let C be the event that arrow ill come to rest in a composite number

$C = \{ 4, 6, 8, 9, 10, 12 \}$ $\Rightarrow n(C) = 6$

$P(C) = \frac{n(C)}{n(S)} \quad = \frac{6}{12} \quad = \frac{1}{2}$

10. ***Write the sample space for tossing three coins using tree diagram.***

11. ***Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6. (using tree diagram).***

Answer:

S = { (1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5)} $\Rightarrow$ **n(S) = 30**

12. ***If A is an event of random experiment such that $P(A) : P(\bar{A}) = 17 : 15$ and $n(S) = 640$, Then find $P(\bar{A})$ and $n(A)$***

Answer:

Given $P(A) : P(\bar{A}) = 17 : 15$

$\Rightarrow \frac{P(A)}{P(\bar{A})} = \frac{17}{15}$ $\quad \Rightarrow \frac{1- P(\bar{A})}{P(\bar{A})} = \frac{17}{15}$ $\quad (P(A) + P(\bar{A}) = 1)$

$\Rightarrow 15\,(1 - P(\bar{A})) = 17\,P(\bar{A})$ $\quad \Rightarrow 15 - 15\,P(\bar{A}) = 17\,P(\bar{A})$

$\Rightarrow 15 = 17\,P(\bar{A}) + 15\,P(\bar{A})$ $\quad \Rightarrow 15 = 32\,P(\bar{A})$

$\Rightarrow \boldsymbol{P(\bar{A}) = \frac{15}{32}}$ $\quad$ (therefore $n(\bar{A}) = 15$)

$\Rightarrow n(S) = 32$ $\quad \Rightarrow n(A) + n(\bar{A}) = 32$ $\quad \Rightarrow n(A) = 32 - n(\bar{A})$ $\quad \Rightarrow n(A) = 32 – 15$

$\Rightarrow$ **n(A) = 17**

13. ***A coin is tossed thrice. What is the probability of getting two consecutive tails ?***

Answer:

Sample Space (S) = { HHH, HHT, HTH, THH, TTT, TTH, THT, HTT }

n(S) = 8

Let A = probability of getting two consecutive tails

A = { HTT, TTH, TTT } $\Rightarrow n(A) = 3$ $\quad \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

14. ***At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater that 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?***

Answer:

n(S) = 1000

(i) Let A be the event of getting perfect squares between 500 and 1000

$A = \{ 23^2, 24^2, 25^2, 25^2, 26^2, 27^2, 28^2, 29^2, 30^2, 31^2\}$

$n(A) = 9$

$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$

First players wins the prize = $\frac{9}{1000}$

(ii) When the card which was taken first is not replaced.

$n(S) = 999$

$n(B) = 8$

$P(B) = \frac{n(B)}{n(S)} = \frac{8}{999}$

15. ***A bag contains 12 blue balls and x red balls. If one ball is drawn at random***

(i) what is the probability that it will be a red ball?

(ii) if 8 more red balls put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

Answer:

Given n(R) = x, n(B) = 12

Total number of balls in the bag = x + 12 (x → red, 12 → black)

(i) Let A be the event of getting red balls

$P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+12}$

(ii) If 8 more red balls are added in the bag $\Rightarrow n(S) = x + 12 + 8 \quad \Rightarrow x + 20$

By the given statement in question

$\Rightarrow \frac{x+8}{x+20} = 2\left(\frac{x}{x+12}\right) \qquad \Rightarrow \frac{x+8}{x+20} = \left(\frac{2x}{x+12}\right)$

$\Rightarrow (x+8)(x+12) = 2x(x+20) \qquad \Rightarrow x^2 +12x + 8x + 96 = 2x^2 + 40x$

$\Rightarrow x^2\ 20x + 96 = 2x^2 + 40x \qquad \Rightarrow x^2 + 20x + 96 - 2x^2 - 40x = 0$

$\Rightarrow - x^2 - 20x + 96 = 0 \qquad \Rightarrow x^2 + 20x - 96 = 0$

$\Rightarrow (x + 24) (x - 4) = 0 \qquad \Rightarrow x = 24$ *& 4 (-ve negligible)*

$\Rightarrow$ *Therefore x = 4*

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+12} \qquad \Rightarrow P(A) = \frac{4}{4+12} \qquad \Rightarrow P(A) = \frac{4}{16}$

$\Rightarrow \mathbf{P(A) = \frac{1}{4}}$

16. ***Two unbiased dice are rolled once. Find the probability of getting***

(i) a doublet (equal numbers on both dice) (ii) the product as a prime number

(iii) the sum as a prime number (iv) the sum as 1

Answer:

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} $\Rightarrow$ **n(S) = 36**

(i) Let A be the event of getting a doublet

A = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) } $\Rightarrow n(A) = 6$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$ $\Rightarrow P(A) = \frac{1}{6}$

(ii) Let B the event of getting the product as a prime number

B = { (1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1) } $\Rightarrow n(B) = 6$

$\Rightarrow P(A) = \frac{n(B)}{n(S)} = \frac{6}{36}$ $\Rightarrow P(A) = \frac{1}{6}$

(iii) Let C be the event of getting the sum of numbers on the dice is prime

C = { (1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)}

$\Rightarrow P(C) = \frac{n(C)}{n(S)} = \frac{15}{36}$ $P(C) = \frac{5}{12}$

(iv) With two dice, minimum sum possible = 2

Therefore probability (sum as 1) = 0

17. *Three fair coins are tossed together. Find the probability of getting*
(i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails.

Answer:

Sample Space (S) = { HHH, HHT, HTH, THH, TTT, TTH, THT, HTT }
n(S) = 8

(i) Let A be the event of getting all heads.

A = { HHH} $\Rightarrow n(A) = 1$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) Let B be the event of getting atleast one tail

B = { HHT, HTH, THH, TTT, TTH, THT, HTT } $\Rightarrow n(B) = 7$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) Let C be the event of getting atmost one head.

C = { TTT, TTH, THT, HTT } $\Rightarrow n(C) = 4$

$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iv) Let D be the event of getting atmost two tails.

D = { HHH, HHT, HTH, THH, TTH, THT, HTT } $\Rightarrow n(D) = 7$

$P(D) = \frac{n(d)}{n(S)} = \frac{7}{8}$

18. ***Two dice are numbered 1, 2, 3, 4, 5, 6 and 1,1,2,2, 3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.***

Answer:

S = *{ (1,1),(1,1),(1,2),(1,2),(1,3),(1,3)*
(2,1),(2,1),(2,2),(2,2),(2,3),(2,3)
(3,1),(3,1),(3,2),(3,2),(3,3),(3,3)
(4,1),(4,1),(4,2),(4,2),(4,3),(4,3)
(5,1),(5,1),(5,2),(5,2),(5,3),(5,3)
(6,1),(6,1),(6,2),(6,2),(6,3),(6,3)} ⇒ ***n(S) = 36***

(i) *Let A be the event of getting sum = 2*

A = { (1, 1), (1, 1)} ⇒ *n(A) = 2*

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

(ii) *Let B be the event of getting sum = 3*

B = { (1, 2), (1, 2), (2, 1), (2, 1)} ⇒ *n(B) = 4*

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(iii) *Let C be the event of getting sum = 4*

A = { (1, 3), (1, 3), (2, 2), (2, 2), (3, 1), (3, 1) } ⇒ *n(C) = 6*

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iv) *Let D be the event of getting sum = 5*

A = { (2, 3), (2, 3), (3, 2), (3, 2), (4, 1), (4, 1)} ⇒ *n(D) = 6*

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(v) *Let E be the event of getting sum = 6*

A = { (3, 3), (3, 3), (4, 2). (4, 2), (5, 1), (5, 1)} ⇒ *n(E) = 6*

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vi) *Let F be the event of getting sum = 7*

A = { (4, 3), (4, 3), (5, 2), (5, 2), (6, 1), (6, 1)} ⇒ *n(F) = 6*

$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vii) *Let G be the event of getting sum = 8*

A = { (5, 3), (5, 3), (6, 2), (6, 2)} ⇒ *n(G) = 4*

$$P(G) = \frac{n(G)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(viii) *Let H be the event of getting sum = 9*

A = { (6, 3), (6, 3)} ⇒ *n(H) = 2*

$$P(H) = \frac{n(H)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

19. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) white ball (ii) black or red ball (iii) not white (iv) neither white nor black

Answer:

Sample space (S) = 5 + 6 + 7 + 8 $\Rightarrow$ *n(S) = 26*

(i) Let A be the probability of getting white ball $\Rightarrow$ *n(A) = 6*

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$$

(ii) Let B be the probability of getting Black or Red ball $\Rightarrow$ *n(B) = 8+5 = 13*

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{26} = \frac{1}{2}$$

(i) Let C be the probability of getting not a white ball $\Rightarrow$ *n(C) = 5 + 7 + 8 = 20*

$$P(C) = \frac{n(c)}{n(S)} = \frac{20}{26} = \frac{10}{13}$$

(i) Let D be the probability of getting neither white nor black ball $\Rightarrow$ *n(D) = 5 + 7 = 12*

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{26} = \frac{6}{13}$$

20. In a box there are 20 non – defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ ***then, find the number of defective bulbs.***

Answer:

Let 'x' be the number of defective bulbs

Total number of bulbs= x + 20 $\Rightarrow$ *n(S) = x + 20*

Let A be the event of selecting defective bulbs = x $\Rightarrow$ *n(A) = x*

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8} \quad \Rightarrow \frac{x}{x+20} = \frac{3}{8} \quad \Rightarrow 8(x) = 3(x + 20)$$

$$\Rightarrow 8x = 3x + 60 \quad \Rightarrow 8x - 3x = 60 \quad \Rightarrow 5x = 60 \quad \Rightarrow x = \frac{60}{5}$$

$$\Rightarrow x = 12$$

Therefore number of defective bulbs = 12

21. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card.

Answer:

Removed cards: King and Queen of Diamonds, Queen and Jack of Hearts and Jack and King of Spades

By the data given $\Rightarrow$ *n(S) = 52 – 2 – 2 – 2 = 46*

(i) Let A be the event of selecting clavor card $\Rightarrow$ *n(A) = 13*

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$$

(ii) Let B be the event of selecting Queen of red card

Queen of diamonds and Heards are removed. $\Rightarrow$ *n(B) = 0*

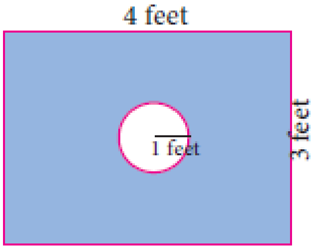
$$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$$

(iii) *Let C be the event of selecting king of black card.*

King of Spade is removed $n(C) = 1$

$\Rightarrow P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$

22. ***Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?***



Answer:

Let A be the probability of win the game = Area of circular region = πr^2 $= \pi(1) = \pi$ *sq. feet*

$\Rightarrow n(A) = \pi$ *sq. feet*

Sample space Area of the rectangular region = l x b = 4 x 3 = 12 sq. feet $\Rightarrow n(S)$ *12 sq. feet*

Probability of win the game $\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{\pi}{12} = \frac{3.14}{12} = \frac{3.14 \times 100}{12 \times 100} = \frac{314}{1200}$

$\Rightarrow$ ***P(A)*** $= \mathbf{\frac{157}{600}}$

23. ***Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on***

(i) the same day (ii) different days (iii) consecutive days

Answer:

Sample space (S) = { Mon, Tue, Wed, Thu, Fri, Sat} $\Rightarrow n(S) = 6$

Probability of Priya and Amuthan to visit shop on any day $= \frac{1}{6}$

(i) *Probability that both of them will visit the shop on same day* $= 6 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6}$

(ii) *Probability that both of them will visit the shop on different days* $= 6 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{6}$

(III) *Probability that both of them will visit the shop on consecutive days*

A = { (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat)} $\Rightarrow n(A) = 5$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$

24. ***If*** $\mathbf{P(A) = 0.37, P(B) = 0.42, P(A \cap B) = 0.09}$ ***then find*** $\mathbf{P(A \cup B)}$

Answer:

Given : $P(A) = 0.37, P(B) = 0.42, P(A \cap B) = 0.09$

$P(AUB) = P(A) + P(B) - P(A \cap B)$ $\Rightarrow P(AUB) = 0.37 + 0.42 - 0.09$ $\Rightarrow P(AUB) = 0.70$

$\Rightarrow P(AUB) = 0.7$

25. ***What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?***

Answer:

Total number of cards = 52 $\Rightarrow n(S) = 52$

Let A be the probability of drawing king cards $\Rightarrow n(A) = 4$

P(A) = $\frac{n(A)}{n(S)}$ = $\frac{4}{52}$ = $\frac{1}{13}$

Let B be the probability of drawing Queen cards $\Rightarrow$ n(B) = 4

P(B) = $\frac{n(B)}{n(S)}$ = $\frac{4}{52}$ = $\frac{1}{13}$

P(A⌒B = 0

P(AUB) = P(A) + P(B) – P(A⌒B) $\Rightarrow$ P(AUB) = $\frac{1}{13}+\frac{1}{13}-0$

$\Rightarrow$ P(AUB) = $\frac{2}{13}$

26. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Answer:

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} $\Rightarrow$ **n(S) = 36**

Let A be the probability of getting doublets

A = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} $\Rightarrow$ n(A) = 6

P(A) = $\frac{n(A)}{n(S)}$ = $\frac{4}{36}$

Let B be the probability of getting face sum 4

B = {(1,3), (2, 2), (3, 1)} $\Rightarrow$ n(B) = 3

P(B) = $\frac{n(B)}{n(S)}$ = $\frac{3}{36}$

(A⌒B) = { (2, 2)} n(A⌒B) = 1

P(A⌒B) = $\frac{n(A\cap B)}{n(S)}$ = $\frac{1}{36}$

P(AUB) = P(A) + P(B) – P(A⌒B) $\Rightarrow$ P(AUB) = $\frac{4}{36}+\frac{3}{36}-\frac{1}{36}$

P(AUB) = $\frac{4+3-1}{36}$ $\Rightarrow$ P(AUB) = $\frac{6}{36}$ **$\Rightarrow$ P(AUB) = $\frac{1}{6}$**

27. If A and B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and P(A and B) = $\frac{1}{8}$, find (i) P (A or B) (ii) P (not A and not B).

Answer:

(i) P(A or B) = P(AUB)

P(AUB) = P(A) + P(B) – P(A⌒C) $\Rightarrow$ P(AUB) = $\frac{1}{4}+\frac{1}{2}-\frac{1}{8}$

P(AUB) = $\frac{2+4-1}{8}$ $\Rightarrow$ P(AUB) = $\frac{5}{8}$

(ii) P (not A and not B).

P (not A and not B) = $P(\bar{A}\cap\bar{B})$

$\Rightarrow P(\bar{A}\cap\bar{B})$ = $P(\overline{AUB})$

= 1 – P(AUB)

$= 1 - \frac{5}{8}$ $\Rightarrow P(\bar{A} \cap \bar{B}) = \frac{8-5}{8}$

$\Rightarrow P(\bar{A} \cap \bar{B}) = \frac{3}{8}$

28. ***A card is drawn from a pack of 52 cards. Find the probability of getting a king or heart or a red card.***

Answer:

Total number of cards = 52 $\Rightarrow n(S) = 52$

Let A be the event of getting a king card $\Rightarrow n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

Let B be the event of getting a heart card $\Rightarrow n(B) = 13$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$

Let C be the event of getting a red card $\Rightarrow n(C) = 13 + 13$ $\Rightarrow n(C) = 26$

$P(C) = \frac{n(c)}{n(S)} = \frac{26}{52}$

$(A \cap B)$ = *Probability of getting heart king* $\Rightarrow n(A \cap B) = 1$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

$(B \cap C)$ = *Probability of getting red and heart* $\Rightarrow n(B \cap C) = 13$

$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{13}{52}$

$(A \cap C)$ = *Probability of getting red king* $\Rightarrow n(A \cap B) = 2$

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{52}$

$(A \cap B \cap C)$ = *Probability of getting heart, king which is red)* $\Rightarrow n(A \cap B \cap C) = 1$

$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{52}$

$\Rightarrow P(AUBUC) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

$= \frac{4+13+26-1-13-2+1}{52}$

$\Rightarrow P(AUBUC) = \frac{28}{52}$ $\Rightarrow P(AUBUC) = \frac{7}{13}$

29. ***In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that***

(i) The student opted for NCC but not NSS. (ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Answer:

Total number of students n(S) = 50

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28 \quad \Rightarrow P(A) = \frac{n(A)}{n(S)} \quad = \frac{28}{50}$

$n(B) = 30 \quad \Rightarrow P(B) = \frac{n(B)}{n(S)} \quad = \frac{30}{50}$

$n(A \cap B) = 18 \quad \Rightarrow P(A \cap B) \quad = \frac{n(A \cap B)}{n(S)} \quad = \frac{18}{50}$

(i) Probability of the students opted for NCC but not NSS

$P(A \cap \bar{B}) \quad = P(A) - P(A \cap B) \quad = \frac{28}{50} - \frac{18}{50} \quad = \frac{28-18}{50} \quad = \frac{10}{50}$

$P(A \cap \bar{B}) \quad = \frac{10}{50}$

(ii) Probability of the students opted for NSS but not NCC

$P(\bar{A} \cap B) \quad = P(B) - P(A \cap B) \quad = \frac{30}{50} - \frac{18}{50} \quad = \frac{30-18}{50} \quad = \frac{12}{50}$

$P(\bar{A} \cap B) \quad = \frac{12}{50}$

(iii) Probability of the students opted for exactly one of them.

$P\,[(A \cap \bar{B}) U (\bar{A} \cap B)] = P(A \cap \bar{B}) + P(\bar{A} \cap B) \quad = \frac{10}{50} + \frac{12}{50} \quad = \frac{10+12}{50} \quad = \frac{22}{50}$

$P\,[(A \cap \bar{B}) U (\bar{A} \cap B)] = \frac{11}{25}$

(note that $(A \cap \bar{B}), (\bar{A} \cap B)$ are mutually exclusive events)

30. A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Answer:

$P(A) = 0.5, \quad P(A \cap B) = 0.3$

We have $P(AUB) \leq 1$

$\Rightarrow P(A) + P(B) - P(A \cap B) \leq 1$

$\Rightarrow 0.5 + P(B) - 0.3 \leq$

$\Rightarrow 0.2 + P(B) \leq 1 \qquad \Rightarrow P(B) \leq 1 - 0.2$

$\Rightarrow P(B) \leq 0.8$

Therefore, probability of B getting selected is atmost 0.8

31. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(AUB) = \frac{1}{3}$ then find $P(A \cap B)$

Answer:

$P(AUB) = P(A) + P(B) - P(A \cap B) \qquad \Rightarrow \frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A \cap B)$

$\Rightarrow -P(A \cap B) = \frac{1}{3} - \frac{2}{3} - \frac{2}{5} \qquad \Rightarrow P(A \cap B) = -\frac{1}{3} + \frac{2}{3} + \frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{-5+10+6}{15} \qquad \Rightarrow P(A \cap B) = \frac{11}{15}$

32. A and B are two events such that, P(A) = 0.42, P(B) = 0.48 and P $(A \cap B)$ = 0.16. Find

(i) P (not A) (ii) P (not B) (iii) P (A or B)

Answer:

Given : P(A) = 0.42, P(B) = 0.48 and P $(A \cap B)$ = 0.16.

(i) P (not A) $= P(\bar{A})$ = 1 – P(A)

= 1 – 0.42

= 0.58

(ii) P (not B) $= P(\bar{B})$ = 1 – P(B)

= 1 – 0.48

= 0.52

(iii) P (A or B) = P(AUB) = P(A) + P(B) – $P(A\cap B)$

= 0.42 + 0.48 – 0.16

= 0.74

33. ***If A and B are two mutually exclusive events of a random experiment and P (not A) = 0.45, P (A U B) = 0.65, then find P(B)***

Answer:

Given : P (not A) = 0.45, P (A U B) = 0.65

A and B are mutually exclusive $\Rightarrow P(A\cap B) = 0$

$\Rightarrow$ P(A) = 1 - $P(\bar{A})$ $\Rightarrow$ P(A) = 1 – 0.45

$\Rightarrow$ P(A) = 0.55

$\Rightarrow$ P(AUB) = P(A) + P(B) – $P(A\cap B)$ $\Rightarrow$ 0.65 = 0.55 + P(B) – 0

$\Rightarrow$ P(B) = 0.65 – 0.55 $\Rightarrow$ P(B) = 0.10

$\Rightarrow$ P(B) = 0.1

34. ***The probability that atleast one of A or B occur is 0.6. If A and B occur simultaneouosly with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.***

Answer:

Given : P(AUB) = 0.6, $P(A\cap B) = 0.2$

$P(\bar{A}) + P(\bar{B})$ = [1 – P(AUB)] + [1 – $P(A\cap B)$]

= (1 – 0.6) + (1 – 0.2)

= 0.4 + 0.8

= 1.2

35. ***The proabability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.***

Answer:

Given : P(A) = 0.5, P(B) = 0.3

$\Rightarrow$ P(AUB) = P(A) + P(B) – $P(A\cap B)$ $\Rightarrow$ P(AUB) = 0.5 +0.3 + 0

$\Rightarrow$ P(AUB) = 0.8

Probability that neither A nor B

$\Rightarrow P(\overline{AUB})$ = 1 – P(AUB)

$\Rightarrow P(\overline{AUB})$ = 1 – 0.8

$\Rightarrow P(\overline{AUB})$ = 0.2

Probability that neither A nor B is 0.2

36. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Answer:

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} ⇒ ***n(S) = 36***

Let A be the event of getting even number on the 1st die

A = *{ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)*
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} ⇒ *n(A) = 18*

$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

Let B be the probability of getting face sum 8

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} ⇒ *n(B) = 5*

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$(A \cap B) = \{ (2, 6), (4, 4), (6, 2)\}$ ⇒ $n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{3}{36}$

$P(AUB) = P(A) + P(B) - P(A \cap C)$ ⇒ $P(AUB) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$P(AUB) = \frac{18+5-3}{36}$ ⇒ $P(AUB) = \frac{20}{36}$ ⇒ $\mathbf{P(AUB) = \frac{5}{9}}$

37. From a well – shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Answer:

Total number of cards = 52 ⇒ *n(S) = 52*

Let A be the probability of drawing red king cards ⇒ *n(A) = 2*

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52} = \frac{2}{52}$

Let B be the probability of drawing black Queen cards ⇒ *n(B) = 2*

$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52} = \frac{2}{52}$

$P(A \cap B = 0$

$P(AUB) = P(A) + P(B) - P(A \cap B)$ ⇒ $P(AUB) = \frac{2}{52} + \frac{2}{52} - 0$

⇒ $P(AUB) = \frac{2+2}{52}$

⇒ $P(AUB) = \frac{4}{52}$ ⇒ $P(AUB) = \frac{1}{13}$

38. A box contains cards numbered 3, 5, 7, 9, ………………., 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Answer:

S = { 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31,33, 35, 37 } ⇒ *n(S) = 18*

Let A be the event of multiples of 7 cards = { 7, 21, 35 } $\Rightarrow$ *n(A) = 3*

$$P(A) = \frac{n(A)}{n(S)} \quad = \frac{3}{18}$$

Let B be the event of prime number cards = { 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,37 } $\Rightarrow$ *n(B) = 11*

$$P(B) = \frac{n(B)}{n(S)} \quad = \frac{11}{18}$$

$(A \cap B) = \{7\}$ $\quad n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \quad = \frac{1}{18}$$

$P(AUB) = P(A) + P(B) - P(A \cap B)$ $\Rightarrow$ $P(AUB) = \frac{3}{18} + \frac{11}{18} - \frac{1}{18}$

$\Rightarrow$ $P(AUB) = \frac{3+11-1}{18}$

$\Rightarrow$ $P(AUB) = \frac{13}{18}$

39. ***Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.***

Answer:

Sample Space (S) = { HHH, HHT, HTH, THH, TTT, TTH, THT, HTT }

n(S) = 8

Let A be the event of getting atmost 2 tails.

A = { HHH, HHT, HTH, THH,TTH, THT, HTT } $\Rightarrow$ *n(A) = 7*

$$\Rightarrow P(A) = \frac{n(A)}{n(S)} \quad = \frac{7}{8}$$

Let B be the event of getting atleast 2 heads

B = { HHH, HHT, HTH, THH } $\Rightarrow$ *n(B) = 4*

$$\Rightarrow P(B) = \frac{n(B)}{n(S)} \quad = \frac{4}{8}$$

$(A \cap B)$ *= { HHH, HHT, HTH, THH }* $\Rightarrow n(A \cap B) = 4$

$$\Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{8}$$

$P(AUB) = P(A) + P(B) - P(A \cap B)$ $\Rightarrow$ $P(AUB) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8}$

$\Rightarrow$ $P(AUB) = \frac{7+4-4}{8}$

$\Rightarrow$ $P(AUB) = \frac{7}{8}$

40. ***The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?***

Answer:

Given : $P(A) = \frac{3}{5}, \; P(\bar{B}) = \frac{5}{8}, \; P(AUB) = \frac{5}{7}$

$P(\bar{B}) = \frac{5}{8}$ $\quad \Rightarrow P(B) = 1 - P(\bar{B})$ $\quad \Rightarrow P(B) = 1 - \frac{5}{8}$ $\quad \Rightarrow P(B) = \frac{8-5}{8}$

$\Rightarrow P(B) = \frac{3}{8}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $\Rightarrow$ $P(A\cup B) = \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$

$\Rightarrow$ $P(A\cup B) = \frac{168+105-200}{280}$

$\Rightarrow$ $P(A\cup B) = \frac{73}{280}$

Therefore probability of getting both offer = $\frac{73}{280}$

41. ***In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?***

Answer:

$n(S) = 8000$,

A = Over 50 years $\Rightarrow n(A) = 1300$

B = Females $\Rightarrow n(B) = 3000$

$A\cap B$ = 30% of females $\Rightarrow n(A\cap B) = \frac{30}{100} \times 3000$ $\Rightarrow n(A\cap B) = 30 \times 30$

$\Rightarrow n(A\cap B) = 900$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{1300}{8000}$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{3000}{8000}$

$\Rightarrow P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{900}{8000}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $\Rightarrow$ $P(A\cup B) = \frac{1300}{8000} + \frac{3000}{8000} - \frac{900}{8000}$

$\Rightarrow$ $P(A\cup B) = \frac{1300+3000-900}{8000}$

$\Rightarrow$ $P(A\cup B) = \frac{3400}{8000}$ $\Rightarrow$ $P(A\cup B) = \frac{34}{80}$

$\Rightarrow$ $P(A\cup B) = \frac{17}{40}$

42. ***A coin is tossed thrice. Find the probability of getting exactly two heads or at least one tail or two consecutive heads.***

Answer:

Sample Space (S) = { HHH, HHT, HTH, THH, TTT, TTH, THT, HTT }

$n(S) = 8$

Let A be the event of getting exactly 2 heads.

A = { HHT, HTH, THH } $\Rightarrow n(A) = 3$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

Let B be the event of getting atleast one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT } $\Rightarrow n(B) = 7$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} \quad = \frac{7}{8}$

Let C be the event of getting two consecutive heads

$C = \{HHT, THH, HHH\} \qquad \Rightarrow n(B) = 3$

$\Rightarrow P(C) = \frac{n(C)}{n(S)} \quad = \frac{3}{8}$

$(A \cap B) = \{HHT, HTH, THH\} \qquad \Rightarrow n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$

$(B \cap C) = \{HHT, THH\} \qquad \Rightarrow n(B \cap C) = 2$

$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{2}{8}$

$(A \cap C) = \{HHT, THH\} \quad \Rightarrow n(A \cap C) = 2$

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{8}$

$(A \cap B \cap C) = \{HHT, THH\} \qquad \Rightarrow n(A \cap B \cap C) = 2$

$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{2}{8}$

$\Rightarrow P(AUBUC) \quad = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$

$= \frac{3+7+3-3-2-2+2}{8}$

$\Rightarrow P(AUBUC) \quad = \frac{8}{8} \qquad \Rightarrow P(AUBUC) \quad = 1$

43. If A, B, C are any three events such that probability of B is twice as that of probability of A and B probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $p(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A U B U C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$, then find P (A), P(B) and P(C)?

Answer:

$P(B) = 2P(A)$, $P(C) = 3P(A)$, $P(A \cap B) = \frac{1}{6}$, $p(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A U B U C) = \frac{9}{10}$,

$P(A \cap B \cap C) = \frac{1}{15}$

$\Rightarrow P(AUBUC) \quad = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$\frac{9}{10} \quad = P(A) + 2P(A) + 3P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$

$\frac{9}{10} \quad = 6P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15} \qquad \Rightarrow \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15} = 6P(A)$

$6P(A) = \frac{108+20+30+15-8}{120}$

$6P(A) = \frac{165}{120} \qquad \Rightarrow P(A) = \frac{165}{120} \, x \, \frac{1}{6} \qquad \Rightarrow P(A) = \frac{165}{720} \qquad \Rightarrow P(A) = \frac{11}{48}$

$\Rightarrow P(A) = \frac{11}{48}$

$\Rightarrow P(B) = 2 \, x \, \frac{11}{48} \qquad$ **$P(B) = \frac{11}{24}$**

$\Rightarrow$ P(C) = 3 x $\frac{11}{48}$ **P(C) =** $\mathbf{\frac{11}{16}}$

44. ***In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4 : 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.***

Answer:

$n(S) = 35$, $n(B) : n(G) = 4 : 3$

$\Rightarrow n(B) = 35 \times \frac{4}{7}$ $\Rightarrow n(B) = 5 \times 4$ $\Rightarrow n(B) = 20$

$\Rightarrow n(G) = 35 \times \frac{3}{7}$ $\Rightarrow n(G) = 5 \times 3$ $\Rightarrow n(G) = 15$

Boys = { 1, 2, 3, 4, 5, 6, 7, 8, 9, `10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 }

Girls = { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

Let A be the event of getting boys with prime numbers.

A = { 2, 3, 5, 7, 11, 13, 17, 19 } $\Rightarrow n(A) = 8$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{8}{35}$

Let B be the event of getting girls with composite numbers.

B = { 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35 } $\Rightarrow n(B) = 12$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$

Let C be the event of getting even roll numbers.

C = { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34 } $\Rightarrow n(C) = 17$

$\Rightarrow P(C) = \frac{n(C)}{n(S)} = \frac{17}{35}$

$(A \cap B) = \{\ \}$ $\Rightarrow n(A \cap B) = 0$

$P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{0}{35} = 0$

$(B \cap C) = \{ 22, 24, 26, 28, 30, 32, 34\}$ $\Rightarrow n(B \cap C) = 7$

$P(B \cap C) = \frac{n(B\cap C)}{n(S)} = \frac{7}{35}$

$(A \cap C) = \{ 2 \}$ $\Rightarrow n(A \cap C) = 1$

$P(A \cap C) = \frac{n(A\cap C)}{n(S)} = \frac{1}{35}$

$(A \cap B \cap C) = \{\ \}$ $\Rightarrow n(A \cap B \cap C) = 0$

$P(A \cap B \cap C) = \frac{n(A\cap B\cap C)}{n(S)} = \frac{0}{35} = 0$

$\Rightarrow P(AUBUC) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0$

$= \frac{8+12+17-7-1}{35}$

$\Rightarrow P(AUBUC) = \frac{29}{35}$