UNIVERSAL MATRIC HR SEC SCHOOL

SEDAPALAYAM
TIRUPPUR - 641664
X - MATHS

"TRY TRY AND TRY AGAIN YOU WILL SUCCEED AT LAST"

10TH - MATHS
(ENGLISH MEDIUM)

UNIT - 1

RELATIONS AND FUNCTIONS

(This Study Material Contains all Example and Excercise Question & Answers)

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UNIT - ①.

RELATIONS AND FUNCTIONS

Example (1.1)

(1.1) If $A=\{1,3,5\}$ and $B=\{2,3\}$ then find
i) $A\times B$ and $B\times A$ ii) Is $A\times B = B\times A$? If not why?
iii) S.T $n(A\times B) = n(B\times A) = n(A)\times n(B)$.

Solution:

Given that $A=\{1,3,5\}$ and $B=\{2,3\}$.

i) $A\times B = \{1,3,5\}\times\{2,3\}$

* $A\times B = \{(1,2),(1,3),(3,2),(3,3),(5,2),(5,3)\}$ —— ①

$B\times A = \{2,3\}\times\{1,3,5\}$

* $B\times A = \{(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)\}$ —— ②

ii) From ① and ② weget ① $\neq$ ②.
$\therefore A\times B \neq B\times A$ as $(1,2)\neq(2,1)$ and $(1,3)\neq(3,1)$, ---etc.

iii) $n(A)=3$, $n(B)=2$.
From ① and ② weget

$n(A\times B) = n(B\times A) = 6$ —— ③

$n(A)\times n(B) = 3\times 2 = 6$ —— ④

$n(B)\times n(A) = 2\times 3 = 6$ —— ⑤.

From ③, ④ and ⑤ weget

$$\boxed{n(A\times B) = n(B\times A) = n(A)\cdot\times n(B)}$$

X - std "MATHS"

UNIT = 1.

RELATIONS AND FUNCTIONS

Example (1.1)

(1) If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then find

i) $A \times B$ and $B \times A$ ii) Is $A \times B = B \times A$? If not why?

iii) S.T $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Solution:

Given that $A = \{1, 3, 5\}$ and $B = \{2, 3\}$.

i) $A \times B = \{1, 3, 5\} \times \{2, 3\}$

* $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$ —— (1)

$B \times A = \{2, 3\} \times \{1, 3, 5\}$

* $B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$ —— (2)

ii) From (1) and (2) we get (1) $\neq$ (2).

$\therefore A \times B \neq B \times A$ as $(1, 2) \neq (2, 1)$ and $(1, 3) \neq (3, 1)$, ... etc.

iii) $n(A) = 3$, $n(B) = 2$.

From (1) and (2) we get

$n(A \times B) = n(B \times A) = 6$ —— (3)

$n(A) \times n(B) = 3 \times 2 = 6$ —— (4)

$n(B) \times n(A) = 2 \times 3 = 6$ —— (5).

From (3), (4) and (5) we get

$$\boxed{n(A \times B) = n(B \times A) = n(A) \cdot \times n(B)}$$

1.2) Example (1.2)

If $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$ then find A and B.

Solution:

Given, $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$

$A = \{$ set of all first coordinates of elements of $A \times B \}$

$\therefore \boxed{A = \{3, 5\}}$

$B = \{$ set of all second coordinate of elements of $A \times B \}$

$\therefore \boxed{B = \{2, 4\}}$

1.3) Example (1.3)

Given $A = \{x \in N \mid 1 < x < 4\}$,

$B = \{x \in W \mid 0 \le x < 2\}$

$C = \{x \in N \mid x < 3\}$

Verify that:

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Solution:

$A = \{x \in N \mid 1 < x < 4\} \Rightarrow \boxed{\therefore A = \{2, 3\}}$

$B = \{x \in W \mid 0 \le x < 2\} \Rightarrow \boxed{\therefore B = \{0, 1\}}$

$C = \{x \in N \mid x < 3\} \Longrightarrow \boxed{\therefore C = \{1, 2\}}$

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$= \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\}$ —— (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$\therefore A \times B = \{(2,0), (2,1), (3,0), (3,1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2,1), (2,2), (3,1), (3,2)\}$.

$(A \times B) \cup (A \times C) = \{(2,0), (2,1), (3,0), (3,1)\} \cup \{(2,1), (2,2), (3,1), (3,2)\}$

$= \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\}$ —— ②

From ① and ② we get ① = ②.

$\boxed{A \times (B \cup C) = (A \times B) \cup (A \times C)}$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2,1), (3,1)\}$ ← ③.

$A \times B = \{(2,0), (2,1), (3,0), (3,1)\}$

$A \times C = \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,0), (2,1), (3,0), (3,1)\} \cap \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,1), (3,1)\}$ —— ④.

From ③ and ④ we get.

$\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

Exercise - (1.1). (B. Page : 06)

① Find $A \times B$, $A \times A$ and $B \times A$.

i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ ii) $A = B = \{p, q\}$

iii) $A = \{m, n\}$; $B = \phi$.

$A \times B = \{2, -2, 3\} \times \{1, -4\}$

$= \{(2,1), (2,-4), (-2,1), (-2,-4), (3,1), (3,-4)\}$

$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$

$= \{(2,2), (2,-2), (2,3), (-2,2), (-2,-2), (-2,3), (3,2), (3,-2), (3,3)\}$

$B \times A = \{1, -4\} \times \{2, -2, 3\}$

$= \{(1,2), (1,-2), (1,3), (-4,2), (-4,-2), (-4,3)\}$

ii) $A = B = \{p, q\}$

$A \times B = \{p, q\} \times \{p, q\} = \{(p,p), (p,q), (q,p), (q,q)\}$

$A \times A = \{p, q\} \times \{p, q\} = \{(p,p), (p,q), (q,p), (q,q)\}$

$B \times A = \{p, q\} \times \{p, q\} = \{(p,p), (p,q), (q,p), (q,q)\}$

iii). $A = \{m, n\}$; $B = \phi$. $\phi = \{\}$

$A \times B = \{m, n\} \times \{\} = \{\}$

$A \times A = \{m, n\} \times \{m, n\} = \{(m,m), (m,n), (n,m), (n,n)\}$

$B \times A = \{\} \cap \{m, n\} = \{\}$.

---

(2) Let $A = \{1, 2, 3\}$ and $B = \{x / x$ is a prime number less than $10\}$.

Find $A \times B$ and $B \times A$.

Solution: $A = \{1, 2, 3\}$

$B = \{x / x$ is a prime number less than $10\}$.

$\therefore B = \{2, 3, 5, 7\}$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$.

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$.

$= \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (5,1), (5,2), (5,3), (7,1), (7,2), (7,3)\}$.

---

(3) If $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$.

Find A and B.

Sol: $A = \{$set of all I coordinates of elements of $B \times A\}$.

$\therefore A = \{-2, 0, 3\}$

$B = \{$set of all II coordinates of elements of $B \times A\}$.

$B = \{3, 4\}$

(4) Given that :

$A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$.

To find : $A \times A = (B \times B) \cap (C \times C)$.

* $A \times A = \{5, 6\} \times \{5, 6\} = \{(5,5), (5,6), (6,5), (6,6)\}$ ——— (1)

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$.

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\}$

$(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\}$ ——— (2)

From (1) and (2) weget

$\boxed{A \times A = (B \times B) \cap (C \times C)}$

---

(5). Given:

$A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$, $D = \{1, 3, 5\}$

To find: $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$.

Sol: $A \cap C = \{1, 2, 3\} \cap \{2, 3, 5\} = \{3\}$.

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\} = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\} = \{(3,3), (3,5)\}$ ——— (1).

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1,2), (1,3), (1,5), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3,1), (3,3), (3,5), (4,1), (4,3), (4,5)\}$

$(A \times B) \cap C \times D = \{(3,3), (3,5)\}$ ——— (2).

From (1) and (2) weget

$\boxed{(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)}$

---

6. Given that: $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N \mid 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$

$C = \{3, 5\}$.

Verify that: i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

$B \cup C = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ —— ①

$A \times B = \{0, 1\} \times \{2, 3, 4\} = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{0, 1\} \times \{3, 5\} = \{(0,3), (0,5), (1,3), (1,5)\}$

$(A \times B) \cup (A \times C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ —— ②

From ① and ② we get ① = ②.

$$\boxed{A \times (B \cup C) = (A \times B) \cup (A \times C)}$$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

$B \cap C = \{2, 3, 4\} \cap \{3, 5\}$

$B \cap C = \{3\}$

$A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0,3), (1,3)\}$

$A \times (B \cap C) = \{(0,3), (1,3)\}$ —— ①.

$A \times B = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{(0,3), (0,5), (1,3), (1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,2), \cancel{(0,3)}, (0,4), (1,2), \cancel{(1,3)}, (1,4)\} \cap$
$\{\cancel{(0,3)}, (0,5), \cancel{(1,3)}, (1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,3), (1,3)\}$ —— ②

From ① and ② we get

$$\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$$

iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

$A \cup B = \{0, 1\} \cup \{2, 3, 4\} = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

$(A \cup B) \times C = \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$ $\longrightarrow$ ①

$A \times C = \{0, 1\} \times \{3, 5\} = \{(0,3), (0,5), (1,3), (1,5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\} = \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$

$(A \times C) \cup (B \times C) = \{(0,3), (0,5), (1,3), (1,5)\} \cup$
$\{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$ —— ②

From ① and ② we get

$\boxed{(A \cup B) \times C = (A \times C) \cup (B \times C)}$

---

(7) Given that:

* A = The set of all Natural nos lessthan 8.

$\therefore A = \{1, 2, 3, 4, 5, 6, 7\}$.

* B = The set of all prime nos lessthan 8.

$\therefore B = \{2, 3, 5, 7\}$

* C = The set of even prime number $= \{2\}$

$\therefore C = \{2\}$.

Verify that:

i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

ii) $A \times (B - C) = (A \times B) - (A \times C)$.

i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

$A \cap B = \{1, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} \cap \{\cancel{2}, \cancel{3}, \cancel{5}, \cancel{7}\}$

$A \cap B = \{2, 3, 5, 7\}$.

$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$

$(A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ —— ①.

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$.

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\}$.

$= \{(2,2), (3,2), (5,2), (7,2)\}$

$(A \times C) \cap (B \times C) = \{(2,2), (3,2), (5,2), (7,2)\}$ —— ②.

From ① and ② weget ① = ②.

$$\boxed{(A \cap B) \times C = (A \times C) \cap (B \times C)}$$

ii) $A \times (B - C) = (A \times B) - (A \times C)$.

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7),$
$(4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7),$
$(7,3), (7,5), (7,7)\}$ —— ①

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$A \times B = \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7)$
$(3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7)$
[illegible] $(6,3), (6,5), (6,7)$
$(7,2), (7,3), (7,5), (7,7)\}$ —

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$A \times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$(A \times B) - (A \times C) = \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7)$
$(3,3), (3,5), (3,7), (4,3), (4,5), (4,7),$
$(5,3), (5,5), (5,7), (6,3), (6,5), (6,7)$
$(7,3), (7,5), (7,7)\}$ —— ②

From ① and ② weget

$$\boxed{A \times (B - C) = (A \times B) - (A \times C)}$$

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1.4) **Example (1.4) → Pg-08**

Let $A = \{3, 4, 7, 8\}$ and $B = \{1, 7, 10\}$

which of the following sets are relations from A to B?

i) $R_1 = \{(3,7), (4,7), (7,10), (8,1)\}$.

ii) $R_2 = \{(3,1), (4,12)\}$.

iii) $R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$

Solution:

$A \times B = \{3,4,7,8\} \times \{1,7,10\}$.

$A \times B = \{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)\}$

i) $R_1 = \{(3,7), (4,7), (7,10), (8,1)\}$.

$R_1 \subseteq A \times B$.

* $\therefore R_1$ is a relation from A to B.

ii) $R_2 = \{(3,1), (4,12)\}$.

Here, $(4,12) \in R_2$. But $(4,12) \notin A \times B$.

* $\therefore R_2$ is not a relation from A to B.

iii) $R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$

Here, $(7,8) \in R_3$. But $(7,8) \notin A \times B$.

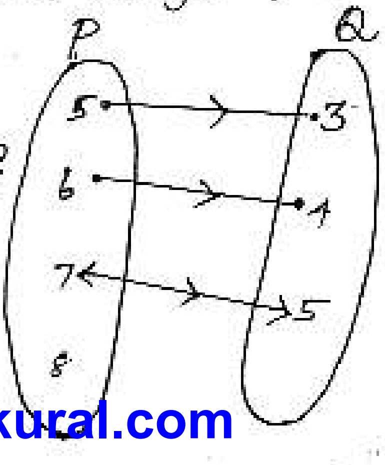
* $\therefore R_3$ is not a relation from A to B.

1.5) **Example (1.5).**

The arrow diagram shows a relationship between the sets P and Q. write the relation in

i) Set builder form ii) Roster form.

iii) what is the domain and range of R.



Solution:

i) Set builder form of R.

$R = \{(x,y) / y = x-2, x \in P, y \in Q\}$

ii) Roster form R

$= \{(5,3), (6,4), (7,5)\}$.

iii) Domain of $R = \{5, 6, 7\}$.

Range of $R = \{3, 4, 5\}$

Exercise (1.2) page - 69.

(1). Given that : $A=\{1,2,3,7\}$ and $B=\{3,0,-1,7\}$.
Which of the following are relations from A to B.
i) $R_1=\{(2,1),(7,1)\}$. ii) $R_2=\{(-1,1)\}$.
iii) $R_3=\{(2,-1),(7,7),(1,3)\}$ iv) $R_4=\{(7,-1),(0,3),(3,3),(0,7)\}$.

Solution:

$$A\times B=\{1,2,3,7\}\times\{3,0,-1,7\}$$
$$=\left\{\begin{array}{l}(1,3),(1,0),(1,-1),(1,7),(2,3),(2,0),(2,-1),(2,7)\\(3,3),(3,0),(3,-1),(3,7),(7,3),(7,0),(7,-1),(7,7)\end{array}\right\}$$

i) $R_1=\{(2,1),(7,1)\}$.
Here $(2,1)\in R_1$. But $(2,1)\notin A\times B$.
* $\therefore R_1$ is not a relation from A to B.

ii) $R_2=\{(-1,1)\}$.
Here $(-1,1)\in R_2$. But $(-1,1)\notin R_2$.
* $\therefore R_2$ is not a relation from A to B.

iii). $R_3=\{(2,-1),(7,7),(1,3)\}$.
Here $\boxed{R_3\subseteq A\times B.}$
* $\therefore R_3$ is a relation from A to B.

iv) $R_4=\{(7,-1),(0,3),(3,3),(0,7)\}$.
Here $(0,3)$ and $(0,7)\in R_4$.
But $(0,3)$ and $(0,7)\notin A\times B$.
* $\therefore R_4$ is not a relation from A to B.

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(2). Given: $A = \{1, 2, 3, 4, ------, 45\}$.

$A \times A = \{1, 2, 3, 4, -----, 45\} \times \{1, 2, 3, 4, -----, 45\}$

$\therefore A \times A = \{(1,1), (1,2), (1,3), (1,4), ------, (45,45)\}$.

Then R be the relation defined as "is square of" on A.

$\therefore R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$.

$R \subseteq A \times A$.

* Domain of $R = \{1, 2, 3, 4, 5, 6\}$.
* Range of $R = \{1, 4, 9, 16, 25, 36\}$.

---

(3). Given: $R = \{(x,y) / y = x+3, \; x \in \{0, 1, 2, 3, 4, 5\}\}$.

Here, domain $(x) = \{0, 1, 2, 3, 4, 5\}$.

* Co-domain $(y) = x+3$.

$y_0 = 0+3 = 3$	$(0,3)$
$y_1 = 1+3 = 4$	$(1,4)$
$y_2 = 2+3 = 5$	$(2,5)$
$y_3 = 3+3 = 6$.	$(3,6)$
$y_4 = 4+3 = 7$	$(4,7)$
$y_5 = 5+3 = 8$.	$(5,8)$

* $\therefore R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$.
* Domain $= \{0, 1, 2, 3, 4, 5\}$.
* Range $= \{3, 4, 5, 6, 7, 8\}$.

---

(4) i) Given: $\{(x,y) / x = 2y, \; x \in \{2, 3, 4, 5\}, \; y \in \{1, 2, 3, 4\}\}$

i) To find:

a) Arrow diagram.

b). Graph.

c) A set in roster form.

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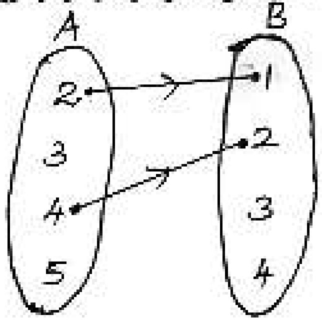
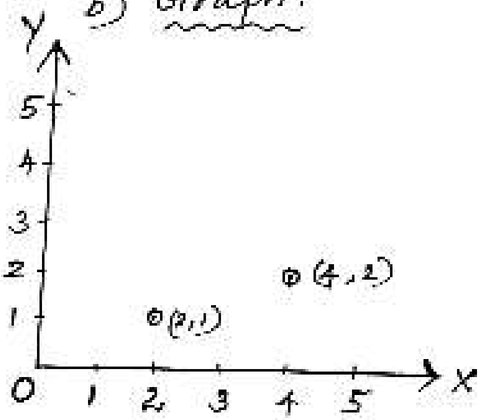
a) Arrow diagram:



b) Graph:



c) A set of roster form:

$R = \{(2,1), (4,2)\}$.

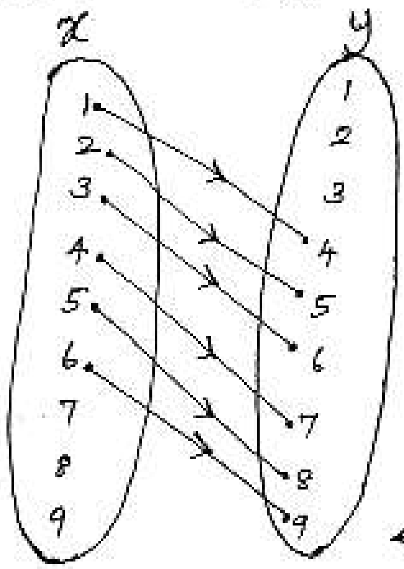
ii). $\{(x,y) / y = x+3,\ x, y$ are natural numbers $< 10\}$.

$\therefore A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

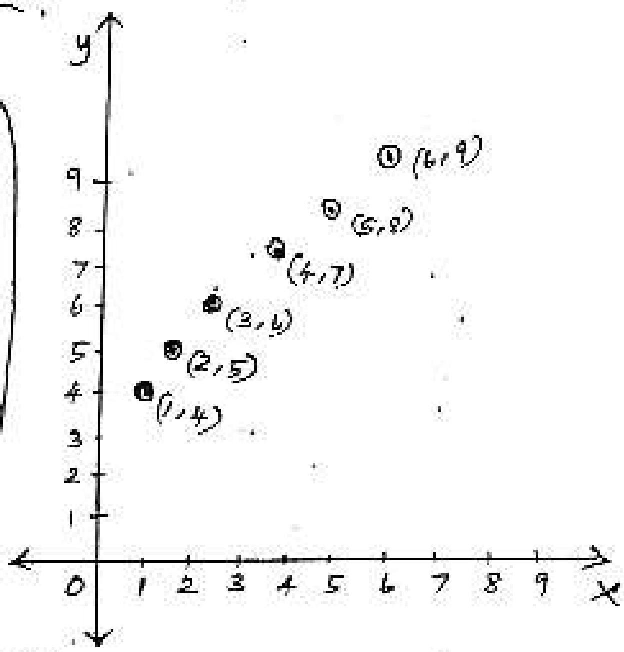
$\boxed{y = x+3}$

$x=1 \Rightarrow y = 1+3 = 4$	$(1,4)$
$x=2 \Rightarrow y = 2+3 = 5$	$(2,5)$
$x=3 \Rightarrow y = 3+3 = 6$	$(3,6)$
$\Rightarrow y = 4+3 = 7$	$(4,7)$
$y = 5+3 = 8$	$(5,8)$
$x=5 \Rightarrow$	
$x=6 \Rightarrow y = 6+3 = 9$	$(6,9)$.

a) An arrow diagram:



b) A graph:



c) A set in roster form:

$R = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

(5) Solution:

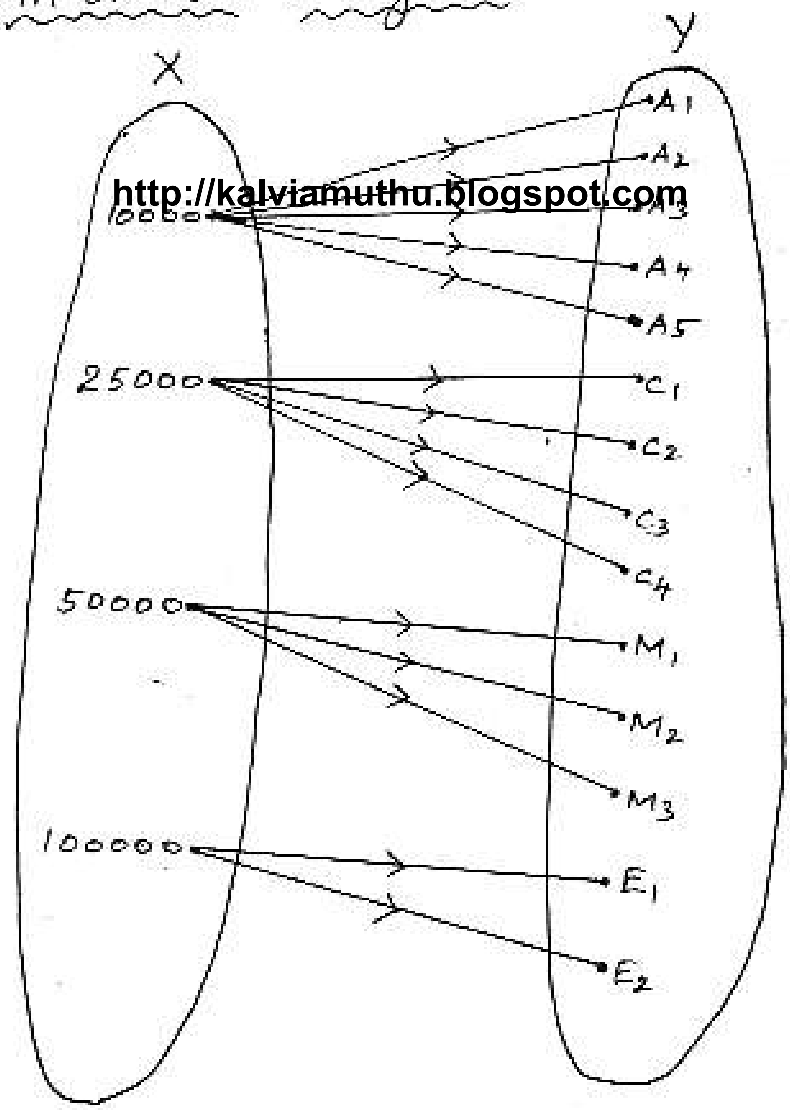
Salaries (S) = { 10000, 25000, 50000, 100000 }

Employees (E) = { A_1, A_2, A_3, A_4, A_5, C_1, C_2, C_3, C_4, M_1, M_2, M_3, E_1, E_2 }

* An ordered pairs:

R = { (10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5)
(25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4)
(50000, M_1), (50000, M_2), (50000, M_3)
(100000, E_1), (100000, F_2) }.

* An arrow diagram:



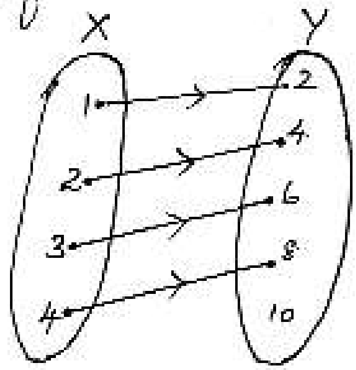
1.6) **Example (1.6) ⇒ Pg-12**

Let $X = \{1,2,3,4\}$ and $Y = \{2,4,6,8,10\}$

$R = \{(1,2),(2,4),(3,6),(4,8)\}$.

To find:

R is a function, domain, co-domain and Range.



For each $x \in X$, there exists only one $y \in Y$.

All $x \in X$ have image $y \in Y$.

$\therefore$ R is a function.

* Domain $= \{1,2,3,4\}$.
* Co-domain $= \{2,4,6,8,10\}$.
* Range $= \{2,4,6,8\}$.

1.7). **Example (1.7).**

Solution:

Given: $f(x) = x^2 - 2$, where $x \in \{-2,-1,0,3\}$

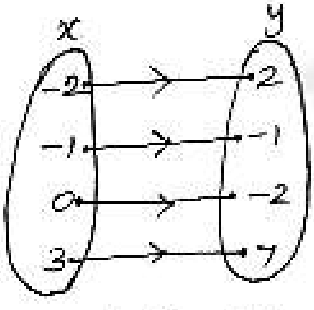
To find: i) The elements of f.

ii) f is a function

$f(x) = x^2 - 2$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$	$(-2,2)$.
$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$	$(-1,-1)$
$f(0) = (0)^2 - 2 = 0 - 2 = -2$	$(0,-2)$
$f(3) = (3)^2 - 2 = 9 - 2 = 7$	$(3,7)$.

$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$.



All element in the domain of f has unique image.

$\therefore f$ is a function.

1.8) **Example (1.8).**

If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$ then which of the following relations are functions from X to Y?

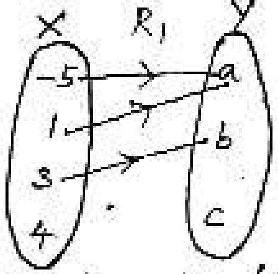
i) $R_1 = \{(-5, a), (1, a), (3, b)\}$

ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$.

iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, a)\}$.

Solution:

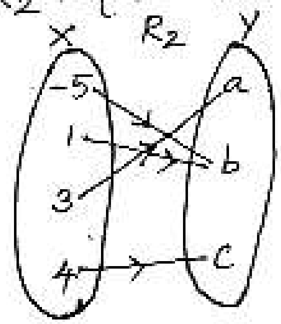
* i). $R_1 = \{(-5, a), (1, a), (3, b)\}$.



* $4 \in X$ in domain there is no image in Y.

$\therefore R_1$ is not a function.

* ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$.

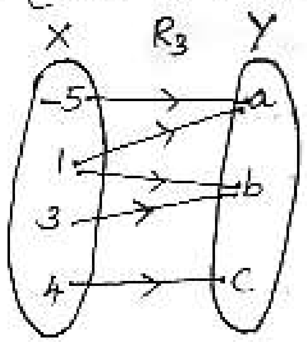


* All elements in X have unique image in Y.

* $\therefore R_2$ is a function.

(P-15)

iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$.



* $1 \in X$ has two images $a \in Y$ and $b \in Y$.
* $\therefore$ R_3 is not a function.

1.9) Example (1.9).

Given: $f(x) = 2x - x^2$.

To find: i) $f(1)$ ii) $f(x+1)$. iii) $f(x) + f(1)$

Let $f(x) = 2x - x^2$

i) $f(1)$ $\quad f(1) = 2(1) - (1)^2$ (Replacing x with 1)

$= 2(1) - 1$

* $\boxed{f(1) = 1}$

ii) $f(x+1)$.

$f(x) = 2x - x^2$

$f(x+1) = 2(x+1) - (x+1)^2$

(Replace $x \Rightarrow x+1$)

$= 2x+2 - [(x^2 + 2x + 1)]$

$f(x+1) = 2x + 2 - (x^2 + 2x + 1)$

$f(x+1) = \cancel{2x} + 2 - x^2 - \cancel{2x} - 1$.

* $\boxed{f(x+1) = -x^2 + 1}$

iii) $f(x) + f(1)$.

$f(x) + f(1) = (2x - x^2) + 1$

$= 2x - x^2 + 1$

* $\boxed{\therefore f(x) + f(1) = -x^2 + 2x + 1}$

Exercise ⇒ 1.3 : pg-13.

1. Let $f = \{(x, y) \mid x, y \in N \text{ and } y = 2x\}$ be a relation on N. Find the domain, co-domain, and range. Is this relation a function?

Solution:

Given $y = 2x$

If $x = 1 \Rightarrow y = 2 \times 1 = 2$

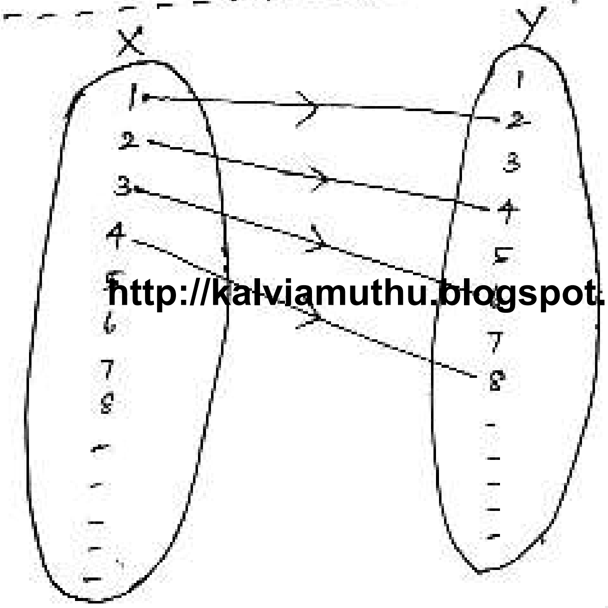
$x = 2 \Rightarrow y = 2 \times 2 = 4$

$x = 3 \Rightarrow y = 2 \times 3 = 6$

$x = 4 \Rightarrow y = 2 \times 4 = 8$

- - - - - - - - - - -

(1, 2).
(2, 4).
(3, 6)
(4, 8).
- - -



$\therefore f = \{(1, 2), (2, 4), (3, 6), (4, 8), \ldots\}$

Domain $= \{1, 2, 3, 4, \ldots\}$

Codomain $= \{1, 2, 3, 4, \ldots\}$

Range $= \{2, 4, 6, 8, \ldots\}$

All elements X in domain has unique image in Y.

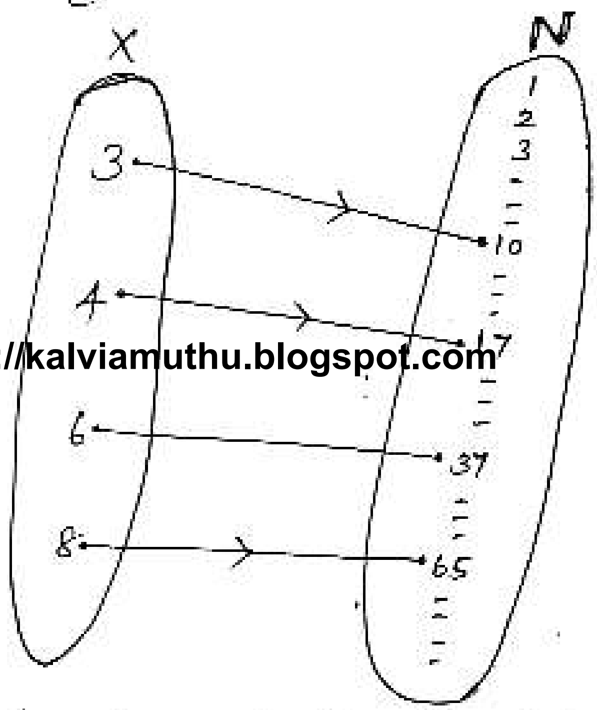
$\therefore$ It is a function.

(2) **Given:** $X = \{3, 4, 6, 8\}$.

$R = \{(x, f(x)) \mid x \in X,\ f(x) = x^2+1\}$.

$f(x) = x^2+1$ |
$f(3) = (3)^2+1 = 9+1 = 10.$ | $(3, 10)$.
$f(4) = 4^2+1 = 16+1 = 17$ | $(4, 17)$.
$f(6) = 6^2+1 = 36+1 = 37$ | $(6, 37)$.
$f(8) = 8^2+1 = 64+1 = 65$ | $(8, 65)$.

$\therefore R = \{(3,10), (4,17), (6,37), (8,65)\}$.



* Each element is domains of X has a unique image in **N**.

$\therefore$ It is a function from X to N.

(3) **Given:** $f : x \rightarrow x^2 - 5x + 6$

To find: i) $f(-1)$ ii) $f(2a)$ iii) $f(2)$ iv) $f(x-1)$

i) $f(-1)$.

$f(x) = x^2 - 5x + 6.$

$f(-1) = (-1)^2 - 5(-1) + 6.$

$= 1 + 5 + 6$

$\boxed{f(-1) = 12}$

$\left[\begin{array}{l}(-1)^2 = -1 \times -1 \\ \quad = +1\end{array}\right]$

ii) $f(2a)$.

$f(x) = x^2 - 5x + 6$.

$f(2a) = (2a)^2 - 5(2a) + 6$.

$= 4a^2 - 10a + 6$.

$\boxed{f(2a) = 4a^2 - 10a + 6}$

$\boxed{(ab)^m = a^m b^m}$

iii) $f(2)$.

$f(x) = x^2 - 5x + 6$

$f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 10 - 10$

$\boxed{f(2) = 0}$

iv) $f(x-1)$.

$\boxed{\begin{array}{l}(a-b)^2 = a^2 - 2ab + b^2 \\ a(b-c) = ab - ac\end{array}}$

$f(x) = x^2 - 5x + 6$.

$f(x-1) = (x-1)^2 - 5(x-1) + 6$.

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

$\boxed{f(x-1) = x^2 - 7x + 12}$

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4. Solution:

i) a) $f(0) = 9$, b) $f(7) = 6$

c) $f(2) = 6$, d) $f(10) = 0$.

ii) $f(x) = 1 \Rightarrow \boxed{x = 9.5}$

iii) a) Domain $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (or)

$= \{x / 0 \le x \le 10, x \in R\}$.

b) Range $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (or)

$= \{x / 0 \le x \le 9, x \in R\}$.

iv) The image of 6 under $f = 5$

(5) Given: $f(x) = 2x+5$.

To find: $\dfrac{f(x+2) - f(2)}{x}$.

$f(x+2) = 2(x+2)+5 = 2x+4+5$

* $\boxed{f(x+2) = 2x+9}$

$f(2) = 2(2)+5 = 4+5$

* $\boxed{f(2) = 9}$

$\therefore \dfrac{f(x+2) - f(2)}{x} = \dfrac{2x+\cancel{9}-\cancel{9}}{x}$

$= \dfrac{2\cancel{x}}{\cancel{x}}$

* $\boxed{\dfrac{f(x+2) - f(2)}{x} = 2}$

(6) Given: $f(x) = 2x-3$

To find:

i) $\dfrac{f(0)+f(1)}{2}$ ii) find x such that $f(x)=0$

iii) x such that $f(x)=x$

iv) x such that $f(x) = f(1-x)$.

i) $\dfrac{f(0)+f(1)}{2} = \dfrac{[2(0)-3]+[2(1)-3]}{2}$

$= \dfrac{0-3+2-3}{2}$

$= \dfrac{-3-1}{2}$

$= \dfrac{-\cancel{4}^{2}}{\cancel{2}_{1}}$

$= -2$

ii) $f(x) = 0$

$2x - 3 = 0$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

iii) $f(x) = x$

$2x - 3 = x$

$2x - x = 3$

$\boxed{x = 3}$

iv) $f(x) = f(1-x)$

$2x - 3 = 2(1-x) - 3$

$2x - 3 = 2 - 2x - 3$

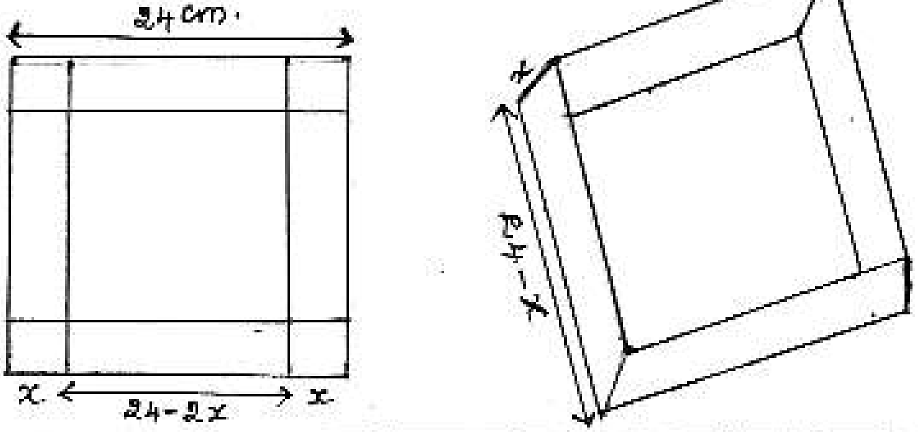
$2x + 2x = 2 - \cancel{3} + \cancel{3}$

$4x = 2$

$x = \frac{\cancel{2}^{1}}{\cancel{4}_{2}}$

$\ast \ \boxed{x = \frac{1}{2}}$

7) Solution:



$\therefore \boxed{l = 24 - 2x, \quad b = 24 - 2x, \quad h = x}$

Volume of the box = Volume of cuboid.

$= lbh$ cu. units.

$= (l \times b \times h)$

$= (24-2x) \times (24-2x) \times x$

$= (24-2x)^2 \times x$ $\quad [(a-b)^2 = a^2 - 2ab + b^2]$

$= (576 - 96x + 4x^2)x$

$= 576x - 96x^2 + 4x^3$

$\therefore$ Volume of the box $= 4x^3 - 96x^2 + 576x$

---

(8) Given: $f(x) = 3-2x$

To find: x, $f(x^2) = (f(x))^2$

$3-2x^2 = (3-2x)^2$

$3-2x^2 = 9-12x+4x^2$

$9-12x+4x^2-3+2x^2 = 0$

$6x^2-12x+6 = 0$

$\div$ by $6 \Rightarrow$

$x^2-2x+1 = 0$

$+$	$\times$
-2	1
$\frac{-1}{x}$	$\frac{-1}{x}$

$(x-1)(x-1) = 0$

$(x-1)^2 = 0$

$x-1 = 0$

$* \boxed{x = 1}$

---

(9) Solution:

Speed $= 500$ km/hr

distance $=$ 'd' km.

Time $=$ 't' hr

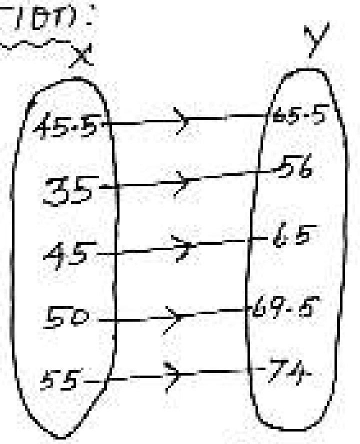
Speed $= \dfrac{\text{distance}}{\text{time}}$

$500 = \dfrac{d}{t}$

$* \boxed{d = 500t}$

(10) Solution:

i)



All elements in x is associated with a unique element in y.

$\therefore$ It is a function.

ii) From the given table. To find: a and b.

When $x = 35$ then $y = 56$.

$$y = ax + b.$$

$$56 = 35a + b$$

$$35a + b = 56 \quad \text{------ } (1).$$

when $x = 45$ then $y = 65$.

$$y = ax + b.$$

$$65 = 45a + b$$

$$45a + b = 65 \quad \text{------ } (2).$$

Solve (1) and (2). we get

$$35a + \cancel{b} = 56 \quad \text{------ } (1)$$

$$\underset{(-)}{45a} \underset{(-)}{+ \cancel{b}} = \underset{(-)}{65} \quad \text{------ } (2).$$

$$(1) - (2) \Rightarrow -10a = -9.$$

$$-10a = -9.$$

$$a = \frac{\cancel{-}9}{\cancel{-}10}$$

$$\boxed{a = \frac{9}{10}} \qquad \boxed{\therefore a = 0.9}$$

Sub $a = 0.9$ in (1) we get

$35a + b = 56$

$35(0.9) + b = 56$

$31.5 + b = 56$

$b = 56 - 31.5$

$\boxed{b = 24.5}$

$$\begin{array}{r} 35 \\ 9 \\ \hline 315 \end{array} *$$

$$\begin{array}{r} \overset{5}{\cancel{5}}\overset{}{\cancel{6}}.\overset{10}{0} \\ (-)31.5 \\ \hline 24.5 \end{array}$$

$\therefore y = ax + b$

$* \boxed{y = 0.9x + 24.5}$

iii) when $x = 40 \Rightarrow y = 0.9x + 24.5$

$y = (0.9 \times 40) + 24.5$

$y = 36.0 + 24.5$

$\boxed{y = 60.5}$

$$\begin{array}{r} 36.0 \\ 24.5 \\ \hline 60.5 \end{array}$$

Height of a women = 60.5 inches.

iv) when $y = 53.3$ then $x = ?$

$y = 0.9x + 24.5$

$53.3 = \underline{0.9x} + 24.5$

$0.9x = 53.3 - 24.5$

$0.9x = 28.8$

$0.9 \times x = 28.8$

$$\begin{array}{r} \overset{4\,12}{\cancel{5}\cancel{3}}.\overset{13}{\cancel{3}} \\ -\ 24.5 \\ \hline 28.8 \end{array}$$

$x = \dfrac{28.8}{0.9}$

$x = \dfrac{28.8}{0.9} \times \dfrac{10}{10}$

$x = \dfrac{288}{9}$

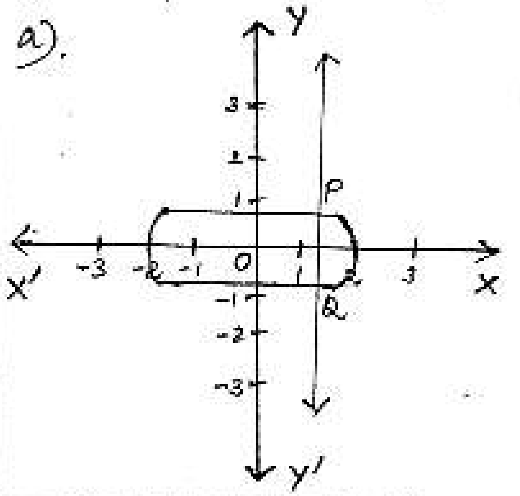
$$\begin{array}{r|l} & 32 \\ \hline 9 & 288 \\ & 27\downarrow \\ \hline & 18 \\ & 18 \\ \hline & 0 \end{array}$$

$* \boxed{x = 32 \text{ cm}}$

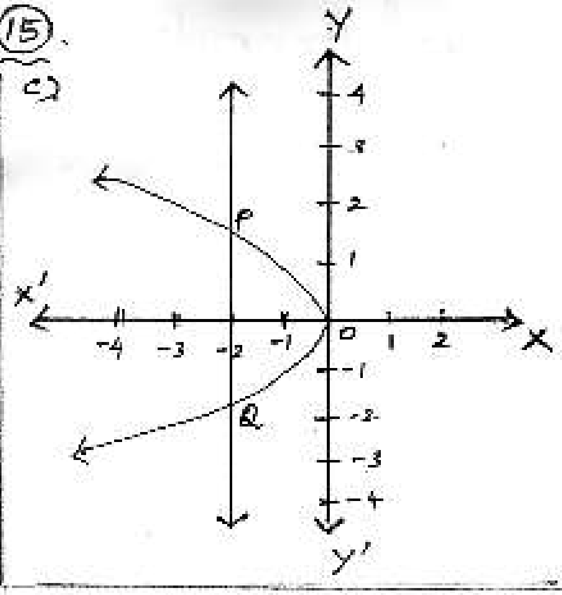
The Length of forehand of a women = 32 cm.

1.10) Example (1.10) ⇒ pg – 15.

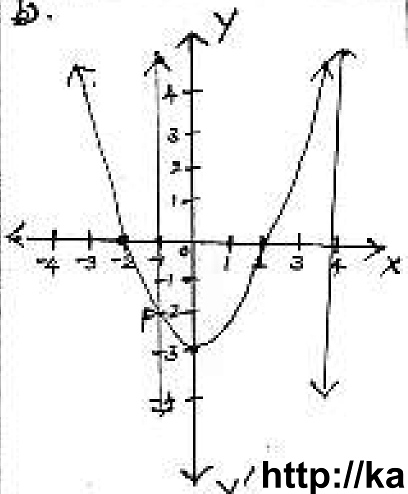
a).

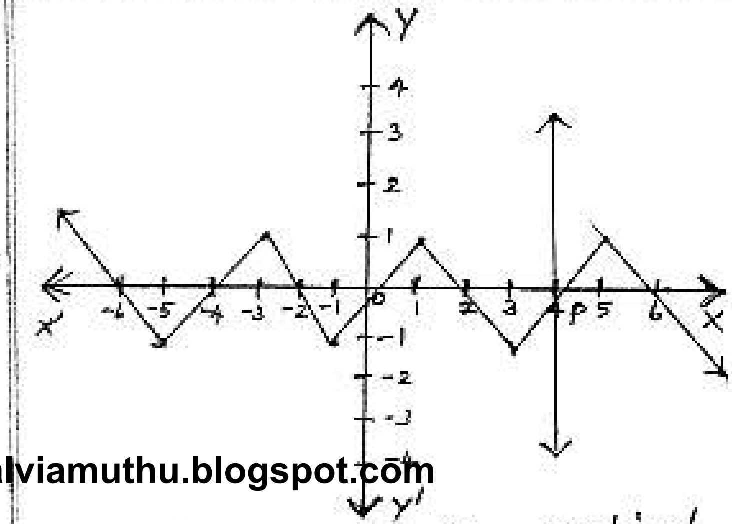


c)



b.





Solution:

i) It is not a function. Because the vertical line meets at more than one point.

ii) It is a function. Because the vertical line meets the curve at one point.

iii) It is not a function. Because the vertical line meets at more than one point.

iv) It is a function. Because the vertical line meets the curve at one point.

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1.11) **Example : (1.11)**

Given that : $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$

Let $f : A \to B$ be a function given by

$$\boxed{f(x) = 3x - 1.}$$

To find:

i) Arrow diagram

ii) Table form

iii) Set of ordered pairs.

iv) Graphical form.

$f(x) = 3x - 1$

$f(1) = 3(1) - 1$

$= 3 - 1$

* $\boxed{f(1) = 2}$

$f(3) = 3(3) - 1$

$= 9 - 1$

* $\boxed{f(3) = 8}$

$f(2) = 3(2) - 1$

$f(2) = 6 - 1$

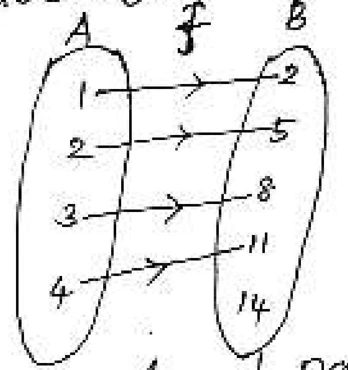
* $\boxed{f(2) = 5}$

$f(4) = 3(4) - 1$

$= 12 - 1$

$\boxed{f(4) = 11}$

i) Arrow diagram:



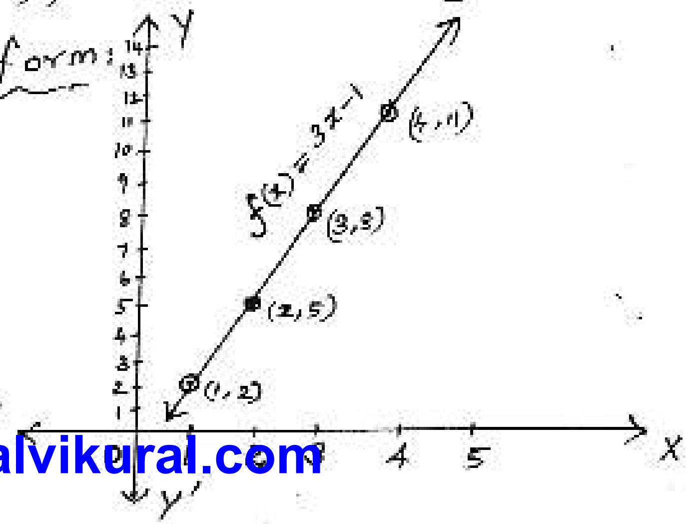
ii) Table form:

x	1	2	3	4
$f(x)$	2	5	8	11

iii) Set of ordered pairs:

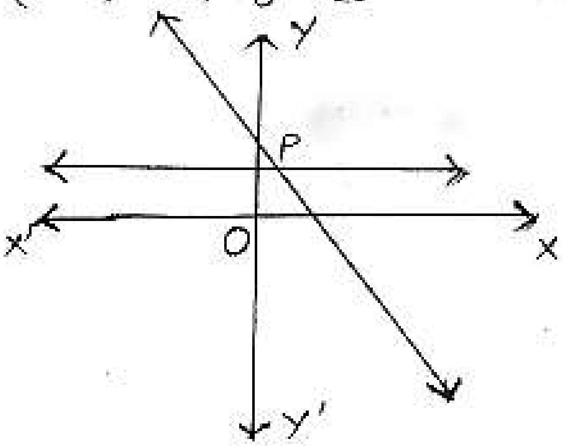
$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}.$

iv) Graphical form:



1.12) Example (1.12) . ⇒ Pg - 20.

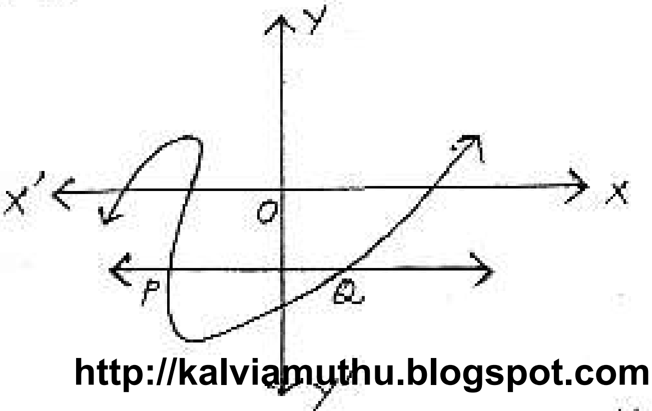
a)



Here the horizonal line meets the curve at one point.

∴ It is one-one function.

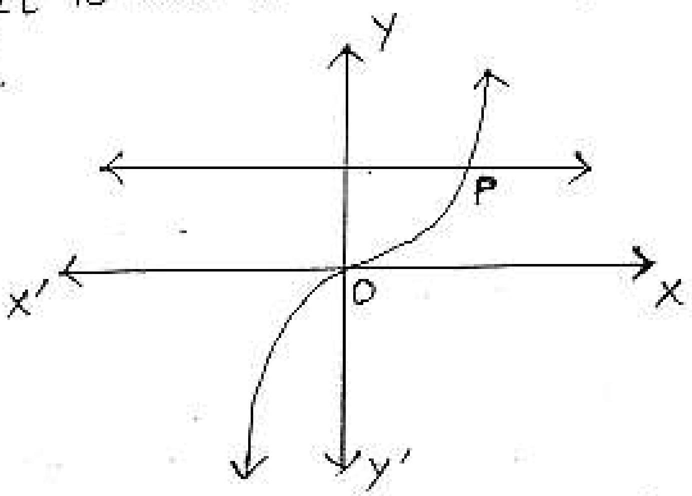
b).



Here the horizontal line meets the curve at more than one point.

It is not a one-one function.

c).



Here the horizontal line meets the curve at One point.

∴ It is One-one function.

1.13) Example : (1.13).

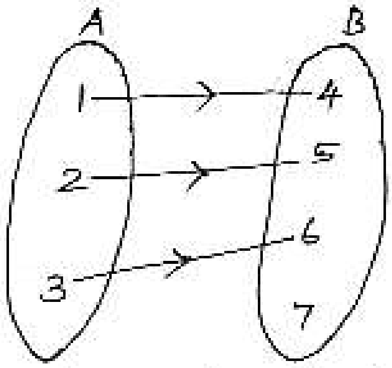
Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6, 7\}$

$f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B. Show that f is one-one but not onto function.

<u>Solution:</u>

Given: $A = \{1, 2, 3\}$ and $B = \{4, 5, 6, 7\}$

$f = \{(1,4), (2,5), (3,6)\}$.



Different elements of A have different

images in B.

$\therefore$ It is one-one function.

The element 7 in co-domain does not have pre-image in domain.

$\therefore$ It is not an onto function.

1.14) Example (1.14)

<u>Given:</u> $A = \{-2, -1, 0, 1, 2\}$.

$f : A \to B$ is an onto function.

$f(x) = x^2 + x + 1$.

$f(x) = x^2 + x + 1$.

$f(-2) = (-2)^2 + (-2) + 1$

$= 4 - 2 + 1$

$= 5 - 2$

$\boxed{f(-2) = 3}$

$f(-1) = (-1)^2 + (-1) + 1$

$= 1 - 1 + 1$

$= \cancel{1} - \cancel{1} + 1$

$\boxed{f(-1) = 1}$

$f(0) = (0)^2 + 0 + 1$

$f(0) = 0 + 0 + 1$

$\boxed{f(0) = 1}$

$f(x) = x^2 + x + 1$

$f(1) = (1)^2 + 1 + 1$

$= 1 + 1 + 1$

$\boxed{f(1) = 3}$

$f(2) = (2)^2 + (2) + 1$

$= 4 + 2 + 1$

$\boxed{f(2) = 7}$

Since f is an onto function.

Range of $f = B =$ Co-domain of f.

$* \therefore \boxed{B = \{1, 3, 7\}}$

1.15) Example (1.15).

Given $f : N \to N$; $f(x) = 3x + 2$

i) $f(x) = 3x + 2$

$f(1) = 3(1) + 2$

$= 3 + 2$

$\boxed{f(1) = 5}$

$f(2) = 3(2) + 2$

$= 6 + 2$

$\boxed{f(2) = 8}$

$f(3) = 3(3) + 2$

$= 9 + 2$

$\boxed{f(3) = 11}$

The images of 1, 2, 3 are 5, 8, 11 respectively.

ii) Pre-image of 29.

$f(x) = 3x + 2$

$29 = 3x + 2$

$3x + 2 = 29$

$3x = 29 - 2$

$3x = 27$

$x = \frac{\cancel{27}^{9}}{\cancel{3}_{1}}$

$\boxed{x = 9}$

Pre-image of 53.

$f(x) = 3x + 2$

$53 = 3x + 2$

$3x + 2 = 53$

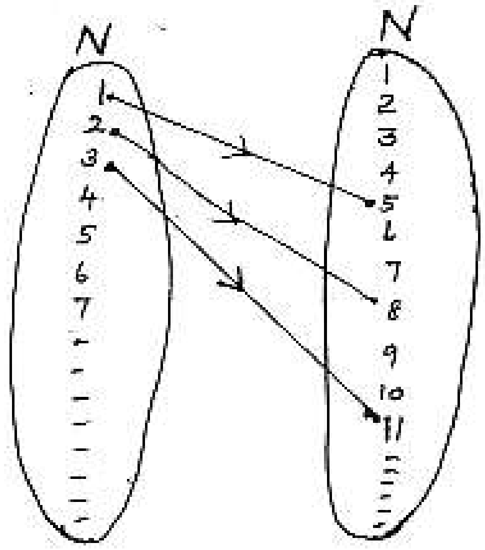
$3x = 53 - 2$

$3x = 51$

$x = \frac{\cancel{51}^{17}}{\cancel{3}_{1}}$

$\boxed{x = 17}$

iii)



* Different elements of N in domain have different image in co-domain.

$\therefore$ It is a one-one function.

Range = $\{5, 8, 11, \ldots\ldots\}$

co-domain = N

Some elements in co-domain does not have pre-image in domain. so it is not onto function.

$\therefore$ It is into function.

1.16) Example (1.16).

Solution: Given $h(b) = 2.47b + 54.10$.

i) $h(b_1) = h(b_2)$.

$2.47b_1 + 54.10 = 2.47b_2 + 54.10$

$2.47b_1 + \cancel{54.10} = 2.47b_2 + \cancel{54.10}$

$\cancel{2.47}b_1 = \cancel{2.47}b_2$

$\boxed{b_1 = b_2}$

$\therefore$ It is one-one function.

ii) Given: $b = 50$, to find $h(b) = ?$

$h(b) = 2.47b + 54.10$

$h(50) = 2.47(50) + 54.10$

$h(50) = (2.47 \times 50) + 54.10$

$h(50) = 123.50 + 54.10$

* $\boxed{h(50) = 177.60 \text{ cm}}$

123.50
(+) 54.10
177.60

iii). height = 147.96 cm.

To find: The length of the thigh bone?

$h(b) = 2.47b + 54.10$

$147.96 = 2.47b + 54.10$

$2.47b = 147.96 - 54.10$

$2.47b = 93.86$

$b = \frac{93.86}{2.47}$

$b = \frac{93.86}{2.47} \times \frac{100}{100}$

$b = \frac{9386}{247}$

* $\boxed{b = 38 \text{ cm}}$

$$\begin{array}{r} 147.96 \\ -\ 54.10 \\ \hline 93.86 \end{array}$$

$$\begin{array}{r|l} & 38 \\ \hline 247 & 9386 \\ & 741 \\ \hline & 1976 \\ & 1976 \\ \hline & 0 \end{array}$$

1.17) Example (1.17).

Given: $f(x) = 3x - 5$

(a, 4) means the image of $a = 4$. $\boxed{\therefore f(a) = 4}$

$f(x) = 3x - 5$

$f(a) = 3a - 5$

$4 = 3a - 5$

$3a - 5 = 4$

$3a = 4 + 5$

$3a = 9$

$a = \frac{\cancel{9}^{3}}{\cancel{3}_{1}}$ * $\boxed{\therefore a = 3}$

(1, b) means the image of $1 = b$. $\boxed{\therefore f(1) = b}$

$f(x) = 3x - 5$

$f(1) = 3(1) - 5$

$b = 3 - 5$

* $\boxed{b = -2}$

1.18) Example (1.18)

Given: $S(t) = \frac{t^2+t}{2}$

i) To find: i) Three and half hours. (3.5).

$S(t) = \frac{t^2+t}{2}$

$S(3.5) = \frac{(3.5)^2+(3.5)}{2}$

$= \frac{(3.5 \times 3.5)+3.5}{2}$

$= \frac{12.25+3.5}{2}$

$S(3.5) = \frac{15.75}{2}$

* $\boxed{S(3.5) = 7.875 \text{ kms}}$

35
35
175
105
1225

12.25
3.5
15.75

7.875
2) 15.75
14
17
16
15
14
10
10
0

ii) Eight hours and fifteen minutes. (8.25)

$S(t) = \frac{t^2+t}{2}$

$S(8.25) = \frac{(8.25)^2+8.25}{2}$

$= \frac{(8.25 \times 8.25)+8.25}{2}$

$= \frac{(68.0625)+8.25}{2}$

$= \frac{68.0625+8.25}{2}$

$S(8.25) = \frac{76.3125}{2}$

* $\boxed{S(8.25) = 38.15625 \text{ kms}}$

825
825
4125
1650
6600
680625

68.0625
8.25
76.3125

38.15625
2) 76.3125
6
16
16
03
2
11
10
12
12
05
4
10
10
0

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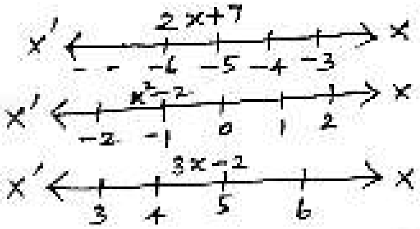
1.19) <u>Example (1.19):</u>

<u>Given:</u>

$$f(x) = \begin{cases} 2x+7, & x < -2 \\ x^2-2, & -2 \le x < 3 \\ 3x-2, & x \ge 3 \end{cases}$$



<u>To find:</u>

i) $f(4)$ ii) $f(-2)$ iii) $f(4)+2f(1)$ iv) $\frac{f(1)-3f(4)}{f(-3)}$.

i) $f(4)$.

Since 4 lies in the interval $x \ge 3$ use $3x-2$.

$f(4) = 3(4)-2$

$= 12-2$

$= 10$

* $\boxed{f(4) = 10}$

ii) $f(-2)$:

Since -2 lies in the interval x^2-2

use x^2-2.

$f(-2) = x^2-2$

$f(-2) = (-2)^2-2$

$[(-2)^2 = -2 \times -2 = +4]$

$= 4-2$

* $\boxed{f(-2) = 2}$

iii) $f(4)+2f(1)$.

Since 1 lies in the interval $-2 \le x < 3$

use x^2-2.

$f(4)+2f(1) = [3(4)-2] + 2[(1)^2-2]$

$= [12-2] + 2[1-2]$

$= 10+2(-1)$

$= 10-2$

* $\boxed{f(4)+2f(1) = 8}$

iv) $\dfrac{f(1) - 3f(4)}{f(-3)}$.

Since -3 lies in the interval $x < -2$ use $2x+7$.

$$\frac{f(1) - 3f(4)}{f(-3)} = \frac{[x^2-2] - 3[3x-2]}{2x+7}$$

$$= \frac{[1^2-2] - 3[3(4)-2]}{2(-3)+7}$$

$$= \frac{[1-2] - 3[12-2]}{-6+7}$$ (∵ by ①) $(f(4) = 10)$

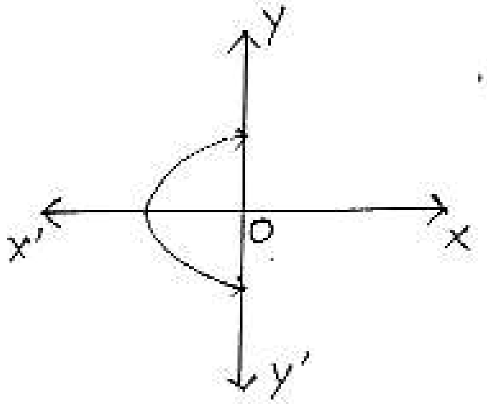
$$= \frac{-1 - 3(10)}{1}$$

$$= -1 - 30$$

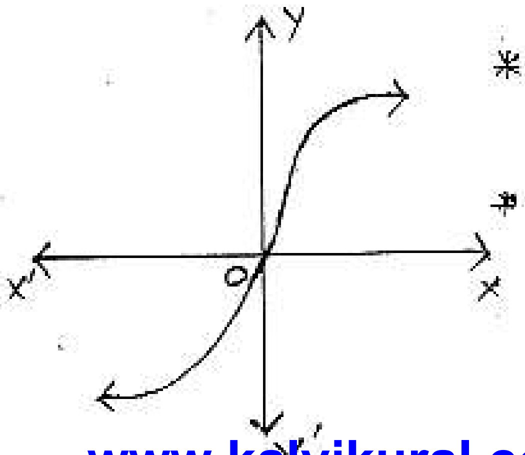
* $$\boxed{\frac{f(1) - 3f(4)}{f(-3)} = -31}$$

Exercise (1.4). ⇒ Pg - 25.

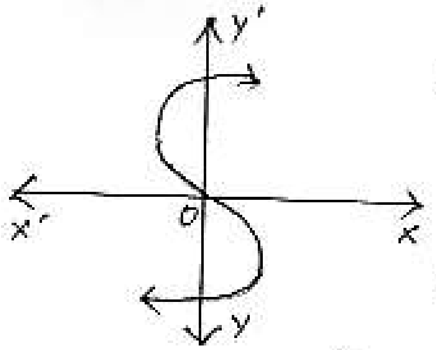
①. i)



* It is not a function.
* Because the vertical line meets at more than one point.

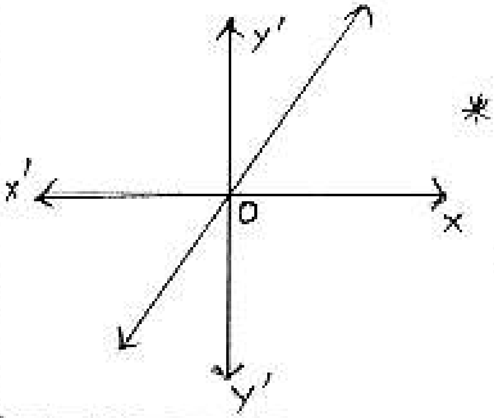


* Vertical line meets the curve at one point.
* It is a function.

iii) 

* Vertical line meets the curve at more than one point.

∴ It is not a function.

iv) 

* Vertical line meets the curve at one point.

∴ It is a function.

(2) Given that:

$A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$.

$f(x) = \frac{x}{2} - 1$

To find:

i) Set of ordered pairs

ii) A table

iii) An arrow diagram

iv) A graph.

$f(x) = \frac{x}{2} - 1$

$f(2) = \frac{\cancel{2}^{1}}{\cancel{2}_{1}} - 1 = 1 - 1 = 0$	$(2, 0)$.
$f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$	$(4, 1)$.
$f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$	$(6, 2)$.
$f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$	$(10, 4)$.
$f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$	$(12, 5)$.

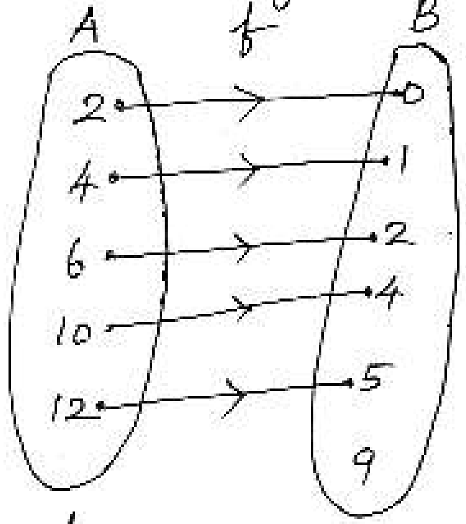
i) Set of ordered pairs:

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$.

ii) A table :

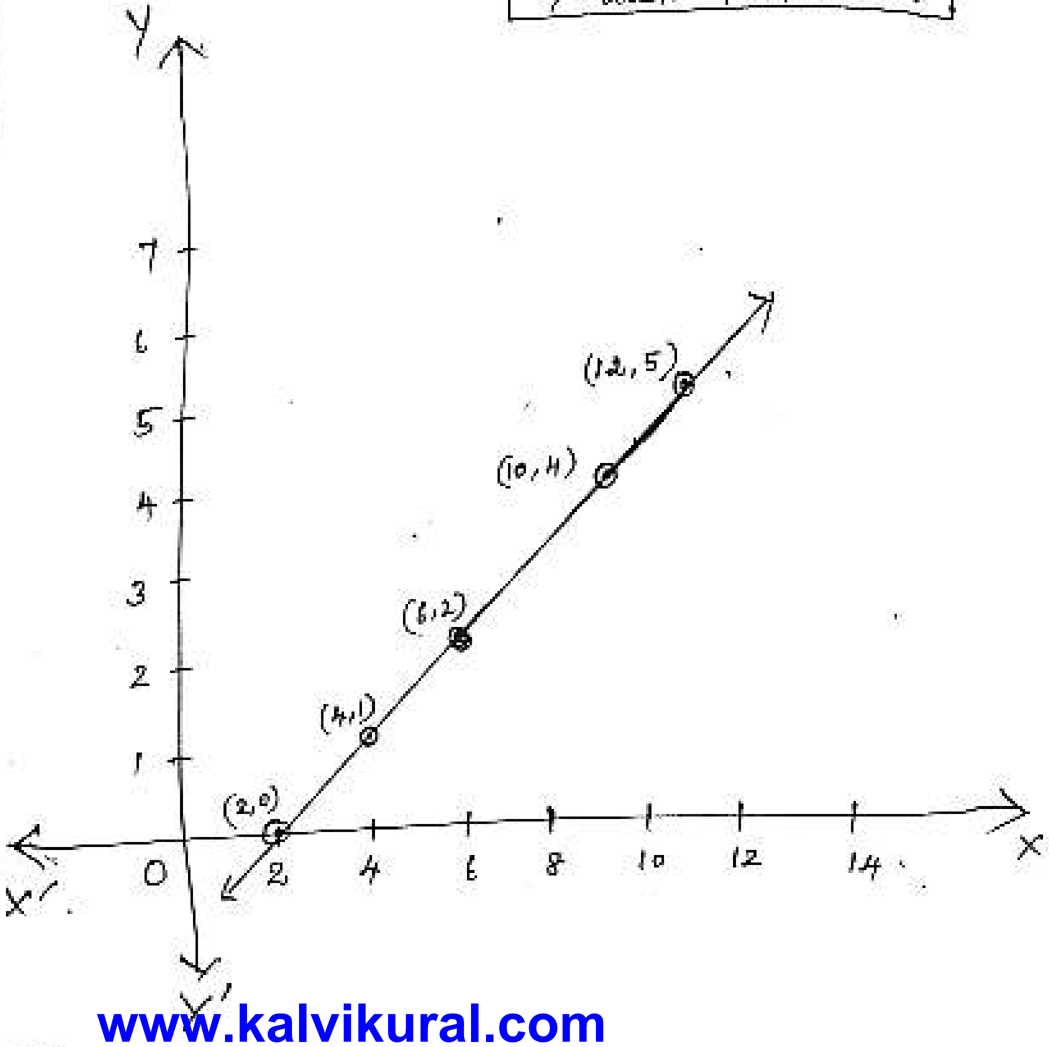
x	2	4	6	10	12
$f(x)$	0	1	2	4	5

iii) An arrow diagram:



iv) A graph:

* Scale *
x-axis 1 cm = 2 units
y-axis 1 cm = 1 unit

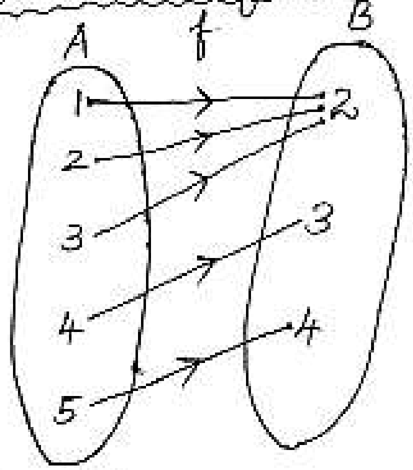


(3) Given: $f = \{(1,2),(2,2),(3,2),(4,3),(5,4)\}$

To find:

i) An arrow diagram ii) A table iii) A graph.

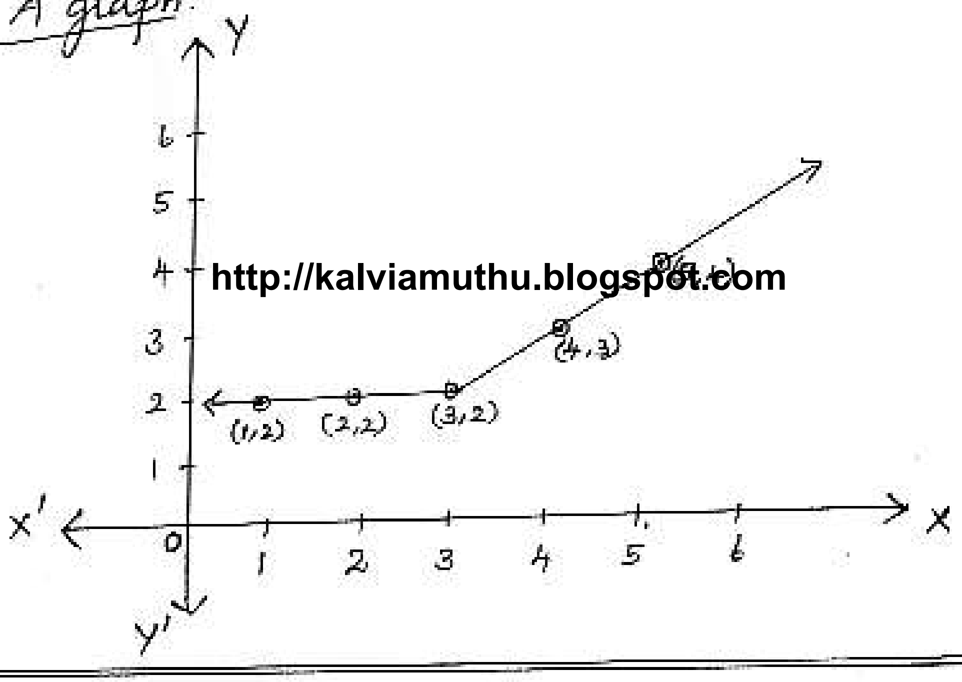
i) An arrow diagram:



ii) A table:

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

iii) A graph:



(4) Given: $f: N \to N$, $f(x) = 2x-1$.

Prove that one-one but not onto function.

Let $f: N \to N$

$N = \{1, 2, 3, 4, 5, \ldots\}$

$f(x) = 2x - 1$

$f(x) = 2x - 1.$

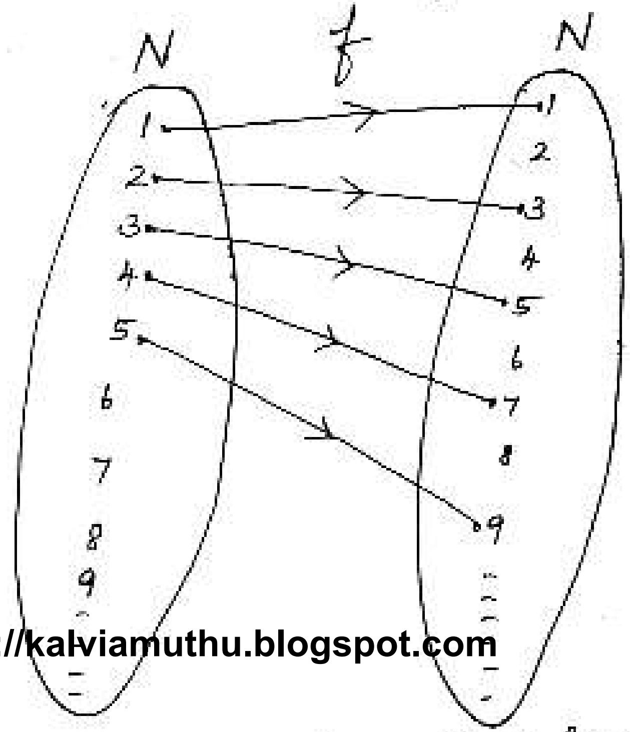
$f(1) = 2(1) - 1$
$= 2 - 1$
$= 1$
$\boxed{f(1) = 1}$

$f(2) = 2(2) - 1$
$= 4 - 1$
$= 3$
$\boxed{f(2) = 3}$

$f(3) = 2(3) - 1$
$= 6 - 1$
$= 5$
$\boxed{f(3) = 5}$

$f(4) = 2(4) - 1$
$= 8 - 1$
$= 7$
$\boxed{f(4) = 7}$

$f(5) = 2(5) - 1$
$= 10 - 1$
$\boxed{f(5) = 9}$



* Different value of x in domain has different image in co-domain.

$\therefore f(x)$ is one-one function.

* $2 \in N$ in co-domain but it does not have pre-image in domain.

$\therefore$ It is not onto function.

(5). Given: $f : N \to N$ $\quad N = \{1, 2, 3, 4, \ldots\}$.

$f(m) = m^2 + m + 3.$

$f(1) = (1)^2 + 1 + 3$
$= 1 + 1 + 3$
$\boxed{f(1) = 5}$

$f(2) = (2)^2 + 1 + 3$
$= 4 + 1 + 3$
$\boxed{f(2) = 8}$

$f(3) = (3)^2 + 3 + 3$

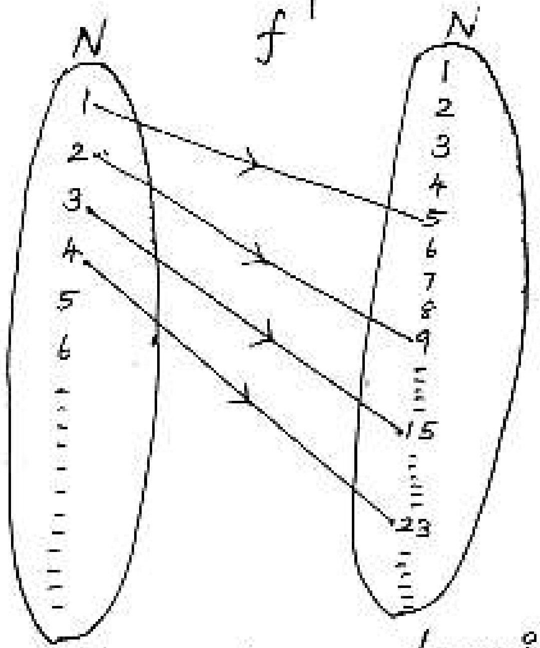
$= 9 + 3 + 3$

$\boxed{f(3) = 15}$

$f(4) = (4)^2 + 4 + 3$

$= 16 + 4 + 3$

$\boxed{f(4) = 23}$



* Distinct element in domain have distinct images in co-domain.

$\therefore$ It is a one-one function.

---

(6) Given: $A = \{1, 2, 3, 4\}$ $B = N = \{1, 2, 3, 4, 5, \ldots\}$

Let $f : A \to B$, $f(x) = x^3$.

$f(x) = x^3$

$f(1) = 1^3 = 1$

$f(2) = 2^3 = 8$

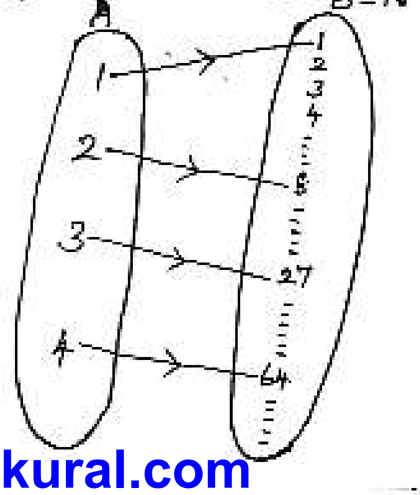
$f(3) = (3)^3 = 27$

$f(4) = (4)^3 = 64$.

$f = \{(1,1), (2,8), (3,27), (4,64)\}$.

i) Range of f:

$= \{1, 8, 27, 64\}$



ii) Distinct elements in A are mapped into distinct images in B.

∴ It is one-one function.

$2 \in N$ in co-domain. But there is no-pre-image in domain.

* So it is a into function.

(7) i) Given $f : R \to R$

i) $f(x) = 2x+1$

$f(0) = 2(0)+1$
$= 0+1$
$\boxed{f(0) = 1}$

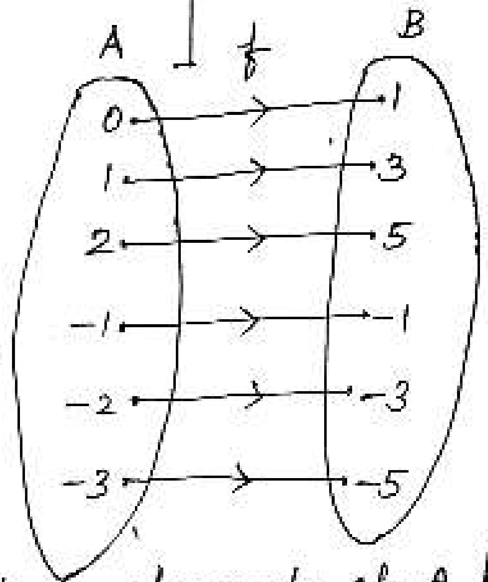
$f(1) = 2(1)+1$
$= 2+1$
$\boxed{f(1) = 3}$

$f(2) = 2(2)+1$
$= 4+1$
$\boxed{f(2) = 5}$

$f(-1) = 2(-1)+1$
$= -2+1$
$\boxed{f(-1) = -1}$

$f(-2) = 2(-2)+1$
$= -4+1$
$\boxed{f(-2) = -3}$

$f(-3) = 2(-3)+1$
$= -6+1$
$\boxed{f(-3) = -5}$



* Distinct elements of A have distinct images in B. It is one-one function.
* Every elements in B has a pre-image in A.

∴ It is an onto function.

∴ It is bijective function.

ii) Given:

$f(x) = 3 - 4x^2$

$f(1) = 3 - 4(1)^2$

$= 3 - 4$

$\boxed{f(1) = -1}$

$f(-1) = 3 - 4(-1)^2$

$= 3 - 4(+1)$

$= 3 - 4$

$\boxed{f(-1) = -1}$

$f(2) = 3 - 4(2)^2$

$= 3 - 4(4)$

$= 3 - 16$

$\boxed{f(2) = -13}$

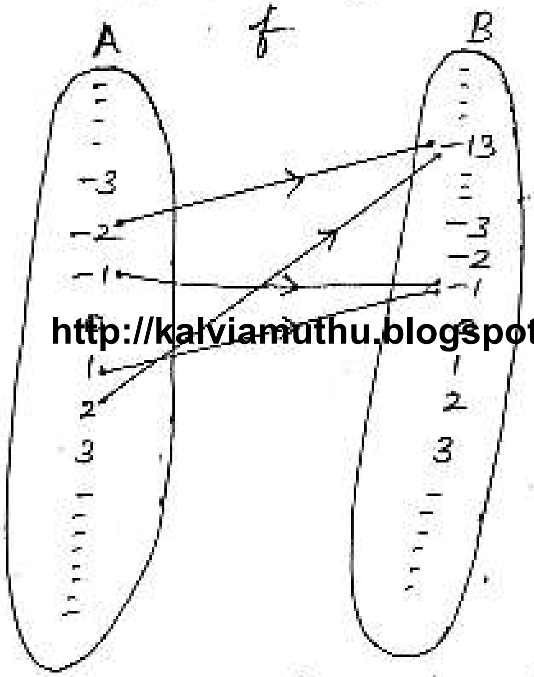
$f(-2) = 3 - 4(-2)^2$

$= 3 - 4(4)$

$= 3 - 16$

$\boxed{f(-2) = -13}$

$(-1)^2 = -1 \times -1$
$= +1$
$(-2)^2 = -2 \times -2$
$= +4$



Two distinct elements 1 and -1 in A have same image -1 in B.

$\therefore$ It is not one-one.

$\therefore$ It is not bijective function.

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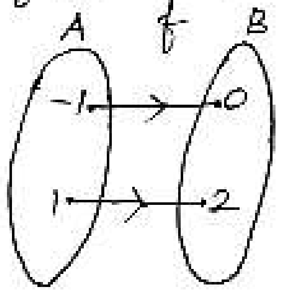
(8) Given that: $A = \{-1, 1\}$ and $B = \{0, 2\}$.

Then $f: A \to B$ defined by $f(x) = ax + b$. is an onto function.



That is, $f(-1) = 0$

$f(1) = 2$

$f(x) = ax + b$

$f(-1) = a(-1) + b = 0 \Rightarrow \quad -a + b = 0. \quad \text{—— (1)}$

$f(1) = a(1) + b = 2 \Rightarrow \quad a + b = 2 \quad \text{—— (2)}$

$-\cancel{a} + b = 0 \quad \text{—— (1)}$

$\cancel{a} + b = 2 \quad \text{—— (2)}.$

$(1) + (2) \Rightarrow \quad 2b = 2$

$b = \frac{\cancel{2}^{1}}{\cancel{2}_{1}}$

$* \boxed{b = 1}$

Sub $b = 1$ in (2) weget

$a + b = 2$

$a + 1 = 2$

$a = 2 - 1$

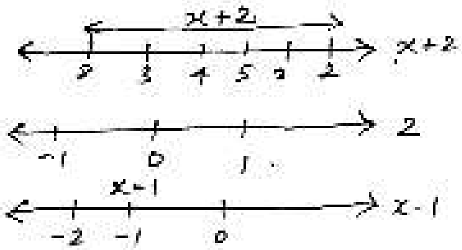
$* \boxed{a = 1}$

Solution

$a = 1$	$b = 1$.

(9) Given:

$$f(x) = \begin{cases} x + 2, & x > 1 \\ 2, & -1 \le x \le 1. \\ x - 1, & -3 < x < -1 \end{cases}$$



To find:

i) $f(3)$. ii) $f(0)$

iii) $f(-1.5)$ iv) $f(2) + f(-2)$.

i) $f(3)$:

Since 3 lies in the interval $x>1$ use $x+2$.

$$f(3) = 3+2$$

* $\boxed{f(3) = 5}$

ii) $f(0)$:

Since 0 lies in the interval $-1 \le x \le 1$ use 2.

* $\boxed{f(0) = 2}$

iii) $f(-1.5)$:

Since -1.5 lies in the interval $-3 < x < -1$.

use $x-1$.

$$f(-1.5) = x-1$$
$$= -1.5-1$$

* $\boxed{f(-1.5) = -2.5}$

iv) $f(2)+f(-2)$.

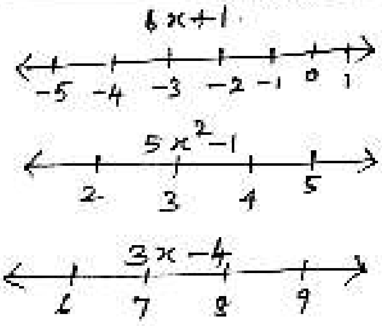
$$f(2)+f(-2) = (x+2)+(x-1)$$
$$= (2+2)+(-2-1)$$
$$= (4)+(-3)$$
$$= 4-3$$

* $\boxed{f(2)+f(-2) = 1}$

---

(10). Given:

$$f(x) = \begin{cases} 6x+1 & , -5 \le x < 2 \\ 5x^2-1 & , 2 \le x < 6 \\ 3x-4 & , 6 \le x \le 9 \end{cases}$$



To find:

i) $f(-3)+f(2)$.

Since -3 lies in the interval $-5 \le x < 2$

use $6x+1$.

$$f(-3) = 6x+1 = 6(-3)+1.$$

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$f(-3) = -18+1$

$\boxed{f(-3) = -17}$

$f(2)$.

since 2 lies in the interval $2 \le x < 6$ use $5x^2-1$.

$f(2) = 5x^2-1$

$= 5(2)^2-1$

$= 5(4)-1$

$= 20-1$

$\boxed{f(2) = 19}$

$\therefore f(-3)+f(2) = -17+19$

* $\boxed{f(-3)+f(2) = 2}$

ii) $f(7)-f(1)$

$f(7)-f(1) = [3x-4] - [6x+1]$

$= (3(7)-4) - (6(1)+1)$

$= [21-4] - [6+1]$

$= 17-7$

* $\boxed{f(7)-f(1) = 10}$

iii) $2f(4)+f(8)$

$2f(4)+f(8) = 2[5x^2-1] + [3x-4]$

$= 2[5(4)^2-1] + [3(8)-4]$

$= 2[5(16)-1] + [24-4]$

$= 2[80-1] + 20$

$= 2(79)+20$ $\quad [f(4) = 79]$

$= 158+20$

* $\boxed{2f(4)+f(8) = 178}$

iv) $\dfrac{2\,f(-2) - f(6)}{f(4) + f(-2)}$

$$\frac{2\,f(-2) - f(6)}{f(4) + f(-2)} = \frac{2[6x+1] - [3x-4]}{[5x^2-1] + [6x+1]}$$

$$= \frac{2[6(-2)+1] - [3(6)-4]}{[5(4)^2-1] + [6(-2)+1]}$$

$$= \frac{2[-12+1] - [18-4]}{[(5\times16)-1] + [-12+1]}$$

$$= \frac{2(-11) - 14}{(80-1) + (-12+1)}$$

$$= \frac{-22-14}{79-11}$$

$$= \frac{-36}{68} = \frac{-9\times\cancel{4}}{17\times\cancel{4}}$$

$$* \boxed{\frac{2f(-2) - f(6)}{f(4) - f(-2)} = \frac{-9}{17}}$$

(11) Given: $S(t) = \frac{1}{2}gt^2 + at + b$

Take: $t = 1, 2, 3, \ldots\ldots$ seconds.

$S(t) = \frac{1}{2}gt^2 + at + b$

$S(1) = \frac{1}{2}g(1)^2 + a(1) + b$

$S(1) = \frac{1}{2}g + a + b$

$\boxed{S(1) = \frac{1}{2}g + a + b}$

$S(t) = \frac{1}{2}gt^2 + at + b$

$S(2) = \frac{1}{2}g(2)^2 + a(2) + b$

$S(2) = \frac{\cancel{4}^{2}}{\cancel{2}_{1}}g + 2a + b$

$\boxed{S(2) = 2g + 2a + b}$

$S(t) = \frac{1}{2}gt^2 + at + b$.

$S(3) = \frac{1}{2}g(3)^2 + a(3) + b$

$= \frac{1}{2}g(9) + 3a + b$

$= \frac{9}{2}g + 3a + b$.

$\boxed{S_3 = 4.5g + 3a + b}$

Distinct elements A have distinct image in B.

It is a one-one function.

(12) Given: $t(c) = F$ $\quad [\because F = \frac{9}{5}c + 32]$

$$t(c) = \frac{9}{5}c + 32$$

To find:

i) $t(0)$.

$$t(c) = \frac{9}{5}c + 32$$

$$t(0) = \frac{9}{5}(0) + 32 = 0 + 32$$

* $\boxed{t(0) = 32^\circ F}$

ii) $t(28)$.

$[9 \times 28 = 252]$

$$\begin{array}{r|l} & 50.4 \\ 5 & 252 \\ & 25\downarrow \\ \hline & 020 \\ & 20 \\ \hline & 0 \end{array}$$

$$t(28) = \frac{9}{5}(28) + 32$$

$$= \frac{252}{5} + 32$$

$$= 50.4 + 32$$

* $\boxed{t28 = 82.4 F}$

iii) $t(-10)$.

$$t(-10) = \frac{9}{5}(-10) + 32$$

$$= \left(\frac{-\cancel{90}^{18}}{\cancel{5}_1}\right) + 32$$

$$= -18 + 32$$

* $\boxed{t(-10) = 14^\circ F}$

iv) To find the value of c. when $t(c) = 212$.

$$t(c) = \frac{9}{5}c + 32$$

$$212 = \frac{9}{5}c + 32$$

$$\frac{9}{5}c + 32 = 212$$

$$\frac{9}{5}c = 212 - 32$$

$$\frac{9}{5}c = 180.$$

$$c = \cancel{180}^{20} \times \frac{5}{\cancel{9}_1}$$

* $\boxed{c = 100^\circ c}$

v). The temperature when the celsius value is equal to Fahrenheit value.

$$F = \frac{9}{5}c + 32$$

$\boxed{C = F}$ ($\because$ By given)

$$F = \frac{9}{5}F + 32$$

$$\frac{9}{5}F - F = -32$$

$$\frac{9F - 5F}{5} = -32$$

$$\frac{4F}{5} = -32$$

$$4F = -32 \times 5$$

$$F = \frac{-\cancel{32}^{8} \times 5}{\cancel{4}}$$

$\boxed{F = -40^\circ}$

1.20) Example (1.20)

Given: $f(x) = 2x+1$ and $g(x) = x^2 - 2$.

To find: $f \circ g$ and $g \circ f$.

$f \circ g(x) = f(g(x))$

$= f(x^2 - 2)$

$= 2(x^2 - 2) + 1$

$= 2x^2 - 4 + 1$

$= 2x^2 - 3$

$\therefore \boxed{f \circ g = 2x^2 - 3}$ — ①

$g \circ f(x) = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2 - 2$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$

$\boxed{g \circ f = 4x^2 + 4x - 1}$ — ②

From ① and ② we get

$f \circ g \neq g \circ f$.

1.21) Example (1.21)

Represent the function $f(x)=\sqrt{2x^2-5x+3}$ as a composition of two functions.

Solution:

we set $f_2(x)=2x^2-5x+3$ and $f_1(x)=\sqrt{x}$

$$f(x)=\sqrt{2x^2-5x+3}$$
$$=\sqrt{f_2(x)}$$
$$=f_1(f_2\,x)$$

$$*\ \boxed{f(x)=f_1f_2(x)}$$

1.22) Example (1.22).

Given: $f(x)=3x-2$, $g(x)=2x+k$

To find: The value of 'k'.

$fog(x)=f(g(x))$

$$=f(2x+k)$$
$$=3(2x+k)-2$$
$$fog(x)=6x+3k-2$$

$gof(x)=g(f(x))$

$$=g(3x-2)$$
$$=2(3x-2)+k$$
$$=6x-4+k$$
$$gof(x)=6x-4+k.$$

Given that $fog=gof$

$$6x+3k-2=6x-4+k$$
$$\cancel{6x}+3k-2=\cancel{6x}-4+k$$
$$3k-2=-4+k$$
$$3k-k=-4+2$$
$$2k=-2$$
$$k=\frac{-\cancel{2}^{1}}{\cancel{2}_{1}}$$

$$*\ \boxed{k=-1}$$

1.23) Example (1.23).

Given: $f \circ f(k) = 5$, where $f(k) = 2k-1$.

To find: The value of 'k'.

$$f \circ f(k) = f(f(k))$$
$$= f(2k-1)$$
$$= 2(2k-1)-1$$
$$= 4k-2-1$$
$$f \circ f(k) = 4k-3$$

But given $f \circ f(k) = 5$

$$\therefore 4k-3 = 5$$
$$4k = 5+3$$
$$4k = 8$$
$$k = \frac{\cancel{8}^{2}}{\cancel{4}_{1}}$$
$$* \boxed{k = 2}$$

1.24) Example (1.24).

Given:

$f(x) = 2x+3$, $g(x) = 1-2x$, $h(x) = 3x$.

To prove:

$$f \circ g(x) = f(g(x))$$
$$= f(1-2x)$$
$$= 2(1-2x)+3$$
$$= 2-4x+3$$
$$= 5-4x.$$
$$(f \circ g) \circ h(x) = (f \circ g)(h(x))$$
$$= (f \circ g)(3x)$$
$$= 5-4(3x)$$
$$= 5-12x \longrightarrow (1)$$

$(g \circ h)(x) = g(h(x))$

$= g(3x)$

$= 1 - 2(3x)$

$= 1 - 6x$

$f \circ (g \circ h)(x) = f(1 - 6x)$

$= 2(1 - 6x) + 3$

$= 2 - 12x + 3$

$= 5 - 12x$ ——— ②

From ① and ② we get.

* $\boxed{(f \circ g) \circ h = f \circ (g \circ h)}$

1.25) Example: (1.25).

Given: $f(x) = 3x+1$, $g(x) = x+3$.

To find x if $g f f(x) = f g g(x)$.

* $g f f(x) = g[f\{f(x)\}]$

$= g[f(3x+1)]$

$= g[3(3x+1)+1]$

$= g[9x + 3 + 1]$

$= g(9x+4)$

$= 9x + 4 + 3$

$\boxed{g f f(x) = 9x + 7}$

* $f g g(x) = f[g\{g(x)\}]$

$= f[g(x+3)]$

$fgg(x) = f[(x+3)+3]$

$= f(x+6)$

$= 3(x+6)+1$

$= 3x+18+1$

$fgg(x) = 3x+19$

Given: $gff(x) = fgg(x)$.

$9x+7 = 3x+19$

$9x-3x = 19-7$

$6x = 12$

$x = \frac{\cancel{12}^{2}}{\cancel{6}_{1}}$

* $\boxed{x = 2}$

Exercise (1.3) ⇒ Pg-(32)

① Given: $f(x) = x-6$, $g(x) = x^2$

i) To find: $f \circ g$ and $g \circ f$. check whether $f \circ g = g \circ f$

$f \circ g = f(g(x))$

$= f(x^2)$.

$\boxed{f \circ g = x^2 - 6}$ —— ①

$g \circ f = g(f(x))$

$= g(x-6)$.

$g \circ f = (x-6)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(x-6)^2 = x^2 - 2 \times x \times 6 + 6^2$

$(x-6)^2 = x^2 - 12x + 36$

$\boxed{g \circ f = x^2 - 12x + 36}$ —— ②

From ① and ② we get

* $\boxed{f \circ g \neq g \circ f}$

ii) Given : $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$.

Tofind : fog and gof.

check whether : $fog = gof$.

$fog = f(g(x))$

$= f(2x^2 - 1)$

$\boxed{fog = \frac{2}{2x^2 - 1}}$ —— ①

$gof = g(f(x))$

$= g\left(\frac{2}{x}\right)$

$= 2\left(\frac{2}{x}\right)^2 - 1$

$= 2\left(\frac{4}{x^2}\right) - 1$

$\boxed{gof = \frac{8}{x^2} - 1}$ —— ②

From ① and ② we get

* $\boxed{fog \neq gof}$

iii) Given: $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$.

Tofind: fog and gof.

$fog = f(g(x))$

$= f(3 - x)$

$= \frac{(3-x)+6}{3}$

$= \frac{3 - x + 6}{3}$

$\boxed{fog = \frac{9 - x}{3}}$ —— ①

$gof = g(f(x))$

$= g\left(\frac{x+6}{3}\right)$

$= 3 - \left(\frac{x+6}{3}\right) = \frac{9 - (x+6)}{3}$

$= \frac{9 - x + 6}{3}$

$\boxed{gof = \frac{3 - x}{3}}$ —— ②.

From ① and ② we get $\boxed{fog \neq gof}$

iv) Given $f(x) = 3 + x$, $g(x) = x - 4$

Tofind: fog and gof.

$fog = f(g(x))$

$= f(x - 4)$

$= 3 + x - 4$

$\boxed{fog = x - 1}$ —— ①

$gof = g(f(x))$

$= g(3 + x)$

$= 3 + x - 4$

$= x - 1$

$\boxed{gof = x - 1}$ —— ②.

From ① and ② we get

* $\boxed{fog = gof}$

v) Given: $f(x) = 4x^2 - 1$; $g(x) = 1 + x$

To find: $f \circ g$ and $g \circ f$.

$f \circ g = f(g(x))$

$= f(1+x)$

$= 4(1+x)^2 - 1$

$= 4(1^2 + 2x + x^2) - 1$

$= 4(1 + 2x + x^2) - 1$

$= 4x^2 + 8x + 4 - 1$

$\boxed{f \circ g = 4x^2 + 8x + 3}$ — ①

$g \circ f = g(f(x))$

$= g(4x^2 - 1)$

$= 1 + 4x^2 - 1$

$= \cancel{1} + 4x^2 - \cancel{1}$

$\boxed{g \circ f = 4x^2}$ — ②.

From ① and ② we get

* $\boxed{f \circ g \neq g \circ f}$

---

② i) Given: $f(x) = 3x + 2$; $g(x) = 6x - k$

To find: The value of k such that $f \circ g = g \circ f$.

$f \circ g = f(g(x))$

$= f(6x - k)$

$= 3(6x - k) + 2$

$= 18x - 3k + 2$

$\boxed{f \circ g = 18x - 3k + 2}$

$g \circ f = g(f(x))$

$= g(3x + 2)$

$= 6(3x + 2) - k$

$= 18x + 12 - k$

$\boxed{g \circ f = 18x + 12 - k}$

But, given that $f \circ g = g \circ f$

$\cancel{18x} - 3k + 2 = \cancel{18x} + 12 - k$

$-3k + 2 = 12 - k$

$2 - 12 = -k + 3k$

$-10 = 2k$

$2k = -10$

$k = -\frac{\cancel{10}^{5}}{\cancel{2}_{1}}$

* $\boxed{k = -5}$

-12
2
-10

ii) **Given:** $f(x) = 2x - k$, $g(x) = 4x + 5$.

$$fog = f(g(x))$$
$$= f(4x+5)$$
$$= 2(4x+5) - k$$
$$\boxed{fog = 8x + 10 - k}$$

$$gof = g(f(x))$$
$$= g(2x-k)$$
$$= 4(2x-k) + 5$$
$$gof = 8x - 4k + 5$$
$$\boxed{gof = 8x - 4k + 5}$$

Given that: $fog = gof$

$$\cancel{8x} + 10 - k = \cancel{8x} - 4k + 5$$
$$10 - k = -4k + 5$$
$$4k - k = 5 - 10$$
$$3k = -5$$
$$* \ \boxed{k = \frac{-5}{3}}$$

$$\begin{array}{r} -10 \\ +5 \\ \hline -5 \\ \hline \end{array}$$

3) **Given:** $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$

To prove: $fog = gof = x$

$$fog = f(g(x))$$
$$= f\left(\frac{x+1}{2}\right)$$
$$= 2\left(\frac{x+1}{2}\right) - 1$$
$$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$$
$$= x + \cancel{1} - \cancel{1}$$
$$\boxed{fog = x} \text{ --- (1)}$$

$$gof = g(f(x))$$
$$= g(2x-1)$$
$$= \frac{(2x-1)+1}{2}$$
$$= \frac{2x \cancel{-1} \cancel{+1}}{2}$$
$$= \frac{\cancel{2}x}{\cancel{2}}$$
$$\boxed{gof = x} \text{ --- (2)}$$

From (1) and (2) we get

$$* \ \boxed{fog = gof = x}$$

(14) Given:

i) $f(x) = x^2 - 1$; $g(x) = x - 2$

$g \circ f(a) = 1$.

Tofind: The value 'a'

$g \circ f(a) = 1$

$f(x) = x^2 - 1$

$f(a) = a^2 - 1$

$g(f(a)) = 1$

$g(a^2 - 1) = 1$

$a^2 - 1 - 2 = 1$

$a^2 - 3 = 1$

$a^2 = 1 + 3$

$a^2 = 4$

$a = \sqrt{4}$

* $\boxed{a = \pm 2}$

ii) Given: $f(k) = 2k - 1$ and $f \circ f(k) = 5$.

Tofind: $k = ?$

$f \circ f(k) = 5$

$f(f(k)) = 5$

$f(2k - 1) = 5$

$2(2k - 1) - 1 = 5$

$4k - 2 - 1 = 5$

$4k - 3 = 5$

$4k = 5 + 3$

$4k = 8$

$k = \frac{\cancel{8}^{2}}{\cancel{4}_{1}}$

* $\boxed{k = 2}$

5. Given that:

$f: A \to B$ defined by

$f(x) = 2x+1$.

$g: B \to C$ defined by

$g(x) = x^2$

<u>To find:</u>

The range of $f \circ g$ and $g \circ f$.

$f \circ g(x) = f(g(x))$

$= f(x^2)$

$= 2x^2+1$

$\therefore \boxed{y = 2x^2+1}$

$\therefore$ Range $= \{y / y = 2x^2+1, x \in N\}$

$g \circ f(x) = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2$

$\boxed{y = (2x+1)^2}$

Range $= \{y / y = (2x+1)^2, x \in N\}$

6. Given: $f(x) = x^2 - 1$.

To find: i) $f \circ f$ ii) $f \circ f \circ f$

i) $f \circ f$

$f \circ f = f(f(x))$

$= f(x^2-1)$

$= (x^2-1)^2 - 1$

$= x^4 \cancel{+1} - 2x^2 \cancel{-1}$

$\boxed{f \circ f = x^4 - 2x^2}$ → 1

ii) $f \circ f \circ f = f(f\{f(x)\})$

$= f(f(x^2-1))$

$= f(x^4 - 2x^2)$ [$\because$ by 1]

$= (x^4 - 2x^2)^2 - 1$

$\boxed{f \circ f \circ f = (x^4 - 2x^2)^2 - 1}$

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(7) If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one ?

Solution:

Given $f(x) = x^5$ and $g(x) = x^4$.

$$f \circ f(x) = f(f(x))$$
$$= f(x^5)$$
$$= (x^5)^5$$
$$\boxed{f \circ f(x) = x^{25}}$$

$f \circ f(1) = 1^{25} = 1.$

$f \circ f(2) = 2^{25}$

$f \circ f(3) = 3^{25}$

Since each elements in f have distinct images.

* $\boxed{\therefore f \text{ is one - one function.}}$

Given: $g : R \to R$ defined by $g(x) = x^4$.

$$g \circ g(x) = g(g(x))$$
$$= g(x^4)$$
$$= (x^4)^4$$
$$\boxed{g \circ g(x) = x^{16}}$$

$$\boxed{g \circ g(x) = 16}$$

(8) **Given:**

i) $f(x) = x-1$; $g(x) = 3x+1$; and $h(x) = x^2$

To find: $(f \circ g) \circ h = f \circ (g \circ h)$

$(f \circ g)(x) = f(g(x))$

$= f(3x+1)$

$= 3x + \cancel{1} - \cancel{1}$

$= 3x$

$(f \circ g) \circ h(x) = (f \circ g)(h(x))$

$= (f \circ g)(x^2)$

$= 3x^2$

$\boxed{(f \circ g) \circ h(x) = 3x^2}$ —— ①

$(g \circ h)(x) = g(h(x))$

$= g(x^2)$

$= 3x^2 + 1$

$f \circ (g \circ h)(x) = f(3x^2+1)$

$= 3x^2 + \cancel{1} - \cancel{1}$

$= 3x^2$

$\boxed{f \circ (g \circ h)(x) = 3x^2}$ —— ②

From ① and ② we get

$(f \circ g) \circ h = f \circ (g \circ h)$

ii) **Given:** $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x+4$.

To find: $(f \circ g) \circ h = f \circ (g \circ h)$

$f \circ g(x) = f(g(x))$

$= f(2x)$

$= (2x)^2$

$= 4x^2$

$(f \circ g) \circ h(x) = (f \circ g)\, h(x)$

$= f \circ g\,(x+4)$

$= 4(x+4)^2$

$(f \circ g) \circ h(x) = 4(x+4)^2$ —— ①

$(g \circ h)(x) = g(h(x))$

$= g(x+4)$

$= 2(x+4)$

$f \circ (g \circ h)(x) = f[2(x+4)]$

$= [2(x+4)]^2$

$= 4(x+4)^2$

$f \circ (g \circ h)(x) = 4(x+4)^2$ —— ②

From ① and ② we get

$(f \circ g) \circ h = f \circ (g \circ h)$

iii) Given: $f(x) = x-4$, $g(x) = x^2$, $h(x) = 3x-5$

$(f\circ g)(x) = f(g(x))$
$= f(x^2)$
$= x^2-4$

$(f\circ g)\circ h(x) = (f\circ g)\,h(x)$
$= (f\circ g)(3x-5)$
$= (3x-5)^2-4$

$\boxed{(f\circ g)\circ h(x) = (3x-5)^2-4}$ — (1)

$(g\circ h)(x) = g(h(x))$
$= g(3x-5)$
$= g(3x-5)$
$= (3x-5)^2$

$f\circ(g\circ h)(x) = f(3x-5)^2$

$\boxed{f\circ(g\circ h)(x) = (3x-5)^2-4}$ — (2)

From (1) and (2) weget
$(f\circ g)\circ h = f\circ(g\circ h)$

(9) Let $f = \{(-1,3), (0,-1), (2,-9)\}$ be a linear function from z into z. find $f(x)$.

Solution: Given:
Linear function $f(x) = ax+b$.

Sub $x=-1$ weget $f(-1) = a(-1)+b$
$3 = -a+b$ — (1)

Sub $x=0$ weget $f(0) = a(0)+b$.
$-1 = 0+b$
$\boxed{b=-1}$

Sub $b=-1$ in (1) weget $-a+b = 3$
$-a-1 = 3$
$-a = 3+1$
$-a = 4$
$\boxed{a=-4}$

The form of linear equation is
$f(x) = ax+b$.
$\boxed{f(x) = -4x-1}$

(10) <u>Given:</u> $C(t) = 3(t)$.

$\therefore C(t_1) = 3(t_1) \Rightarrow C(t_2) = 3(t_2)$.

<u>To prove:</u> The function is linear.

$$C(at_1 + bt_2) = 3(at_1 + bt_2)$$
$$= 3at_1 + 3bt_2$$
$$= a(3t_1) + b(3t_2)$$
$$= ac(t_1) + bc(t_2)$$

$$\boxed{C(at_1 + bt_2) = ac(t_1) + bc(t_2)}$$

$\therefore$ Hence it is linear function.

Exercise (1.6) [ONE MARK QUESTIONS:]

Multiple choice questions:

(1). If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is ______

(1) 1 (2) 2 (3) 3 (4) 6

<u>Solution:</u>

Given $n(A \times B) = 6$.

$A = \{1, 3\}$.

$\therefore n(A) = 2$

$n(A \times B) = n(A) \times n(B)$.

$$n(B) = \frac{n(A \times B)}{n(A)}$$

$$= \frac{\cancel{6}^{3}}{\cancel{2}}$$

$\therefore \boxed{n(B) = 3}$

<u>Ans:</u> (3) 3.

(2). $A = \{a, b, q\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ = ______

(1) 8 (2) 20 (3) 12 (4) 16.

<u>Sol:</u>

$A \cup C = \{a, b, q\} \cup \{p, q, r, s\}$

$A \cup C = \{a, b, p, q, r, s\}$

$\boxed{\therefore n(A \cup C) = 6.}$

$n[(A \cup C) \times B] = n(A \cup C) \times n(B)$

$= 6 \times 2$

$= 12$

$$\boxed{n[(A \cup C) \times B] = 12}$$

<u>Ans:</u> (3) 12.

③. If $A=\{1,2\}$, $B=\{1,2,3,4\}$ $C=\{5,6\}$ and $D=\{5,6,7,8\}$ then state which of the following statement is true.

1). $(A\times C)\subset(B\times D)$

2) $(B\times D)\subset(A\times C)$

3) $(A\times B)\subset(A\times D)$

4) $(D\times A)\subset(B\times A)$.

Solution:

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$B\times D=\{(1,5),(1,6),(1,7),(1,8),$
$(2,5),(2,6),(2,7),(2,8),$
$(3,5),(3,6),(3,7),(3,8)\}$

$(A\times C)\subset(B\times D)$ is true

Ans:

1). $(A\times C)\subset(B\times D)$.

④. If there are 1024 relations from a set $A=\{1,2,3,4,5\}$ to a set B, then the number of elements in B is _____

① 3 ② 2 ③ 4 ④ 8

Solution:

No. of relations $=2^{mn}$ —①

$A=\{1,2,3,4,5\}$

$n(A)=5$.

To find: $n(B)$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

$n(A\times B)=n(A)\times n(B)$

$\therefore n(A)\times n(B)=1024$.

From ① becomes.

$2^{mn}=1024$.

$2^{5n}=2^{10}$.

$5n=10$.

$n=\frac{\cancel{10}^{2}}{\cancel{5}_{1}}$

$\boxed{n=2}$

$\begin{bmatrix} m=n(A) \\ n=n(B) \end{bmatrix}$

$n(B)=2$

Ans: 2). 2.

⑤. The range of the relations $R=\{(x,x^2)/x$ is prime number less than $13\}$ is ____

1). $\{2,3,5,7\}$ (2) $\{2,3,5,7,11\}$

3). $\{4,9,25,49,121\}$.

4). $\{1,4,9,25,49,121\}$.

Sol: Prime No. < 13 is $\{2,3,5,7,11\}$.

$f(x)=x^2$

$f(2)=(2)^2=4$

$f(3)=(3)^2=9$

$f(5)=(5)^2=25$

$f(7)=(7)^2=49$.

$f(11)=(11)^2=121$

Range of R:

$=\{4,9,25,49,121\}$.

Ans:

3) $\{4,9,25,49,121\}$.

(16). If the ordered pairs $(a+2, 4)$ and $(5, 2a+b)$ are equal then (a, b) is ______

1). $(2, -2)$ 2) $(5, 1)$

3) $(2, 3)$ 4) $(3, -2)$

$a+2 = 5$

$a = 5-2$

$\boxed{a = 3}$

$4 = 2a+b$

$2a+b = 4$

$2(3)+b = 4$

$6+b = 4$

$b = 4-6$

$\boxed{b = -2}$

Ans: $(a, b) = (3, -2)$.

4) $(3, -2)$.

(17). Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is ______

1) m^n 2) n^m

3) $2^{mn}-1$ 4) 2^{mn}

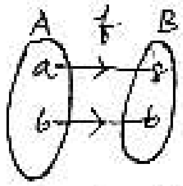
Ans: 3) $2^{mn}-1$

(18). If $\{(a, 8), (6, b)\}$ represents an identity function then the value of a, and b.

1) $(8, 6)$ 2) $(8, 8)$

3) $(6, 8)$ 4) $(6, 6)$.

Sol:

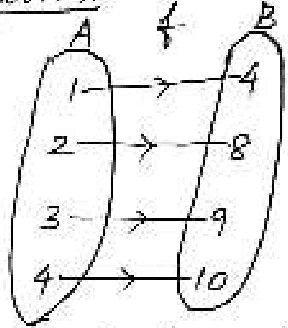


$\therefore (a, b) = (8, 6)$.

Ans: 1) $(8, 6)$.

(19). Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \to B$ given by $f = \{(1,4), (2,8), (3,9), (4,10)\}$ is a ______

Solution:



Distinct elements of A have distinct images in B.

It is one-to one function.

Ans:

3) One-to-one function.

(20). If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$ then fog

Solution:

$f \circ g = f(g(x))$.

$= f\left(\frac{1}{3x}\right)$

$= 2\left(\frac{1}{3x}\right)^2 = 2\left(\frac{1}{9x^2}\right)$

$\boxed{f \circ g = \frac{2}{9x^2}}$ $\left[(ab)^m = a^m b^m\right]$

Ans: 3) $\frac{2}{9x^2}$

(11) If $f: A \to B$ is a bijective function and if $n(B) = 7$ then $n(A) =$ ______

1) 7 2) 49 3) 1 4) 14.

Solution:

Given f is bijective.

$\therefore f$ is one-one and onto function.

$\therefore \boxed{n(A) = 7}$

Ans: 1) 7.

(12) Let f and g be two functions given by

$f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$

$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$.

To find: Range of $f \circ g$

x	$f(x)$	g	$g(x)$
0	1	0	2
2	0	1	0
3	-4	2	4
4	2	-4	2
5	7	7	0

i) $f \circ g(x) = f(g(x))$
$= f(g(0))$
$= f(2)$
$= 0$

ii) $f \circ g(x) = f(g(x))$
$= f(g(1))$
$= f(0)$
$= 2$

iii) $f \circ g(x) = f(g(x))$
$= f(g(2))$
$= f(4)$
$= 0$

iv) $f \circ g(x) = f(g(x))$
$= f(g(-4))$
$= f(2)$
$= 0$

v) $f \circ g(x) = f(g(x))$
$= f(g(7))$
$= f(0)$
$= 1$

* Range of $f \circ g = \{0, 1, 2\}$.

Ans: 4). $\{0, 1, 2\}$

(13). Let $f(x) = \sqrt{1+x^2}$ then ______

put $x = 1$, $y = -1$.

$f(x) = \sqrt{1+x^2}$

$f(x) = f(1) = \sqrt{1+(1)^2} = \sqrt{2}$

$f(y) = f(-1) = \sqrt{1+(-1)^2} = \sqrt{2}$

$xy = (1)(-1) = -1$

$f(xy) = f(-1)$
$= \sqrt{1+(-1)^2}$
$= \sqrt{1+1}$

$f(xy) = \sqrt{2}$

$f(xy) \leq f(x) \cdot f(y)$.

Ans: 3) $f(xy) \leq f(x) \cdot f(y)$

(14) If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the value of α and β. (1) $(-1,2)$ (2) $(2,-1)$ (3) $(-1,-2)$ (4) $(1,2)$.

Solution:

Given $g(x) = \alpha x + \beta$.

Take $\alpha = 2$ and $\beta = -1$.

$g(x) = 2x - 1$.

$g(1) = 2(1) - 1 = 2 - 1 = 1$.

$g(2) = 2(2) - 1 = 4 - 1 = 3$.

$g(3) = 2(3) - 1 = 6 - 1 = 5$

$g(4) = 2(4) - 1 = 8 - 1 = 7$

Ans: 2). $(2,-1)$.

(15) $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is 1) Linear 2) cubic 3) Reciprocal 4) quadratic

$f(x) = (x+1)^3 - (x-1)^3$.

$$\left[\begin{array}{l}(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \\ (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\end{array}\right]$$

$f(x) = [x^3 + 3x^2 + 3x + 1] - (x^3 - 3x^2 + 3x - 1)$

$= \cancel{x^3} + 3x^2 + \cancel{3x} + 1 - \cancel{x^3} + 3x^2 - \cancel{3x} + 1$.

$= 3x^2 + 3x^2 + 1 + 1$

$\boxed{f(x) = 6x^2 + 2}$

It is a quadratic function.

Ans: 4). quadratic.

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